Technical Appendix to Davig and Doh (2008) "Monetary Policy Regime Shifts and Inflation Persistence"

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1. Construction of the Likelihood

Due to regime switching, possible histories of regimes expand as time goes by. Since we apply Kalman filtering conditional on a particular history of regimes, the increasing number of histories makes the filtering process computationally burdensome. Kim and Nelson (1999) fix the maximum number of histories by collapsing M histories to one at each time period if the number of histories exceeds a constant.[1] However, they do not elaborate how to this efficiently. Since certain histories are less plausible than others, treating every history equally may not be efficient. As implemented in Schorfheide (2005), we evaluate the probability of possible histories before collapsing some histories and eliminate, collapse, or preserve each history according to filtered probability of it subject to the constraint that the considered number of histories does not exceed a constant (K).[2]

- Compute $p(Z^t|\vartheta, S^{j,t})$ by applying Kalman filtering on a particular history of regimes $S^{j,t}$.
- Update the probability of each history based on the current observation Z_t.

$$p(S^{j,t}|\vartheta, Z^t) = \frac{p(Z^t|\vartheta, S^{j,t})}{\sum_j p(Z^t|\vartheta, S^{j,t})}$$

- Eliminate history j such that $p(S^{j,t}|\vartheta, Z^t)$ is less than the critical value (0.00001).
- Order the remaining histories based on $p(S^{j,t}|\vartheta, Z^t)$. If the number of eliminated histories is n, keep n histories which have the largest probabilities. Collapse the remaining histories from those which have higher probabilities subject to the constraint that the total number of remaining histories is equal or less than K.

2. Convergence of the Posterior Distribution

We start a MCMC chain initialized at the (local) posterior mode found by a numerical optimization routine. We compute the mean and the covariance matrix based on 1 million draws from the MCMC chain. Then we run another MCMC chain initialized at the mean from the previous chain by using the covariance matrix as the scaling matrix in the proposal density of the Chain. The

[1] M is the number of regimes.

[2] This constant (K) is 8 in a two regime case and 64 in a four regime case.

resulting 100,000 (150,000) draws are used for the posterior inference of the switching monetary policy version (switching variance version and four regime version).

To get an idea that MCMC chains stabilize, we use trace plots of posterior draws of parameters to check the convergence of output from MCMC chains. Figure 1 6 show trace plots of every 100th draw out of 100,000 posterior draws in the three different models. Most parameters show only little to moderate persistence and indicate they behave like being drawn from stationary distributions.

To get a more clear evidence of convergence, we also perform tests of the equality of split sample means. We split 100,000 posterior draws into the two 50,000 subsamples and compute a test statistic based on subsample means and associated standard errors estimated by Newey-West (1987) methods. For most parameters, we could not reject the equality of subsample means at 5% level. Table 1 provide more information on test results.

3. Marginal Likelihood

Marginal likelihood can be computed by the Monte Carlo integration of a probability density function of ϑ as follows.

$$p(Z^T)^{-1} = \int \frac{h(\vartheta)}{p(Z^T|\vartheta)p(\vartheta)} p(\vartheta|Z^T)d\vartheta \longrightarrow \widehat{p}(Z^T)^{-1} = \frac{1}{N}\sum_{i=1} \frac{h(\vartheta_i)}{p(Z^T|\vartheta_i)p(\vartheta_i)}$$

where ϑ_i is a posterior draw. Geweke (1999) proposes an implementation with $h(\cdot)$ a Gaussian density around the posterior mean. Sims, Waggoner, and Zha (2008) show that the Gaussian approximation for the proposal density to compute the marginal likelihood can be misleading and numerically unstable due to the non-Gaussian posterior distribution of parameters in models with regime-switching. They suggest the following elliptical distribution as an alternative.

$$g(\vartheta) = \frac{\Gamma(k/2)}{(2\pi)^{k/2}|det(\overline{S})|}\frac{f(r)}{r^{k-1}} \ , \ r = \sqrt{(\vartheta - \overline{\vartheta})'\overline{\Omega}^{-1}(\vartheta - \overline{\vartheta})} \ , \ \overline{S} = \sqrt{\overline{\Omega}}$$

where $f(r)$ is any one-dimensional density defined on the positive reals which we can estimated based on posterior draws. $\overline{Omega}$ is the sample covariance matrix computed by centering out posterior draws from the posterior mode. Then the weighting function can be constructed as a truncated $g(\cdot)$ where the area with a very low posterior probability is truncated.

$$h(\vartheta) = \frac{\Xi_{\Theta_L}}{q_L} g(\vartheta) \ , \ \Theta_L = \{\vartheta : p(Z^T|\vartheta)p(\vartheta) > L\}$$

where Ξ_{Θ_L} is an indicator function which is 1 if ϑ belong to Θ_L and 0 otherwise. While Sims, Waggoner, and Zha (2008) provide an example where this methods is more robust than Geweke (1999), it does not work well for our purpose. Particularly, $g(\cdot)$ is very sensitive to a few outliers which lead to very low values for r. In contrast, Geweke (1999)'s method provides more reliable estimates of marginal likelihood in our cases. And in the case the marginal likelihood is computed by Sims, Waggoner, Zha (2008)'s method, it is quite similar to the value computed by Geweke (1999)'s method.

4. Prior-Posterior Moments of the Model Implied Persistence

We construct prior distributions of regime dependent parameters to put a positive probability mass at the autocorrelation of inflation in split samples (1953:Q1 - 1979:Q2, 1984:Q1 - 2006:Q4).

Figures 7 - 9 show prior (blue circles) -posterior (green crosses) moments of the model implied persistence based on 200 draws. Split sample autocorrelations are described in red lines. It is noticeable that the passive policy alone can generate the high persistence observed in the first subsample, while high volatility alone cannot generate such high persistence. The low persistence in the second subsample, however, is not completely explained by active monetary policy or low volatility. Importantly, it is the combination of active monetary policy and low volatility in the four regime model that pushes the model implied inflation persistence closer to the sample moment in the second subsample. Our estimation takes full-information likelihood based approach and does not require us to solely match split sample moments because other moments are also taken into account in the estimation. Nonetheless, there is a positive posterior probability mass in the four regime model that one standard error confidence interval of split sample means is covered.

Additional References

Geweke, J. (1999): "Using Simulation Methods for Bayesian Econometric Models: Inference, Development and Communication," *Econometric Reviews*, **18**(1), 1-126.

Newey, W. K., and K. D. West (1987): "A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica* **55**(3), 703-8.

Schorfheide, F. (2005): "Learning and Monetary Policy Shift," *Review of Economic Dynamics*, **8**(2), 392-419.

Sims, C.A.,D.F.Waggoner, and T. Zha (2008): "Methods for inference in large multiple -equation Markov-switching models," *Journal of Econometrics*, forthcoming.

Table 1: Tests of equality of subsample means

Parameters	test statistic			critical value	
	2 MP	2 Variance	2 MP , 2 Variance	5% level	1% level
α		0.1661		1.95996	2.57583
α_1	-0.3788		-0.1910		
α_2	1.0140		-1.2638		
γ		1.4219			
γ_1	-0.3258		-2.2549		
γ_2	-0.4690		-0.7159		
β	-1.4951	-0.3488	-0.3629		
κ	1.4403	-0.2450	-2.8010		
τ	-0.2084	1.6619	-1.7188		
λ	0.3915	-0.5263	-0.2419		
ρ_a	-2.3520	0.2655	1.2362		
ρ_f	-1.7247	1.4594	1.4083		
ρ_i	-2.3515	-0.3564	0.9174		
σ_a	-0.4317				
$\sigma_{a,1}$		-0.5259	0.5075		
$\sigma_{a,2}$		0.4426	0.4820		
σ_u	-0.1896				
$\sigma_{u,1}$		0.1779	1.3293		
$\sigma_{u,2}$		-1.2294	1.4566		
σ_e	-1.0332				
$\sigma_{e,1}$		0.3366	-1.0575		
$\sigma_{e,2}$		0.9003	0.0955		
π^{SS}	-0.0602	-0.7866	0.8214		
$\ln A_0$	0.3492	-0.1040	1.5090		
$\overline{\ln\left(\frac{Y_t^\star}{A_t}\right)}$	-0.2182	0.5957	-2.0921		
p_{11}	0.6056		-0.5136		
p_{22}	1.3021		0.0381		
q_{11}		-2.6421	-1.3121		
q_{22}		-0.5903	-0.8852		

Figure 1: Trace Plots 1 : Switching Policy

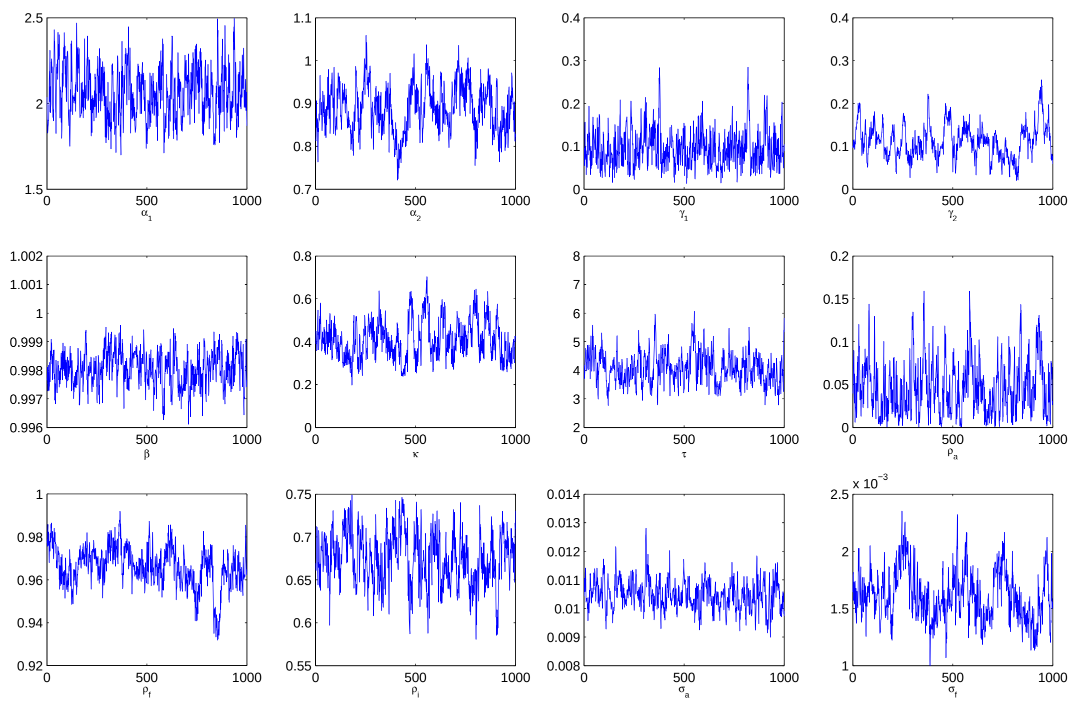

Figure 2: TRACE PLOTS 2 : SWITCHING POLICY

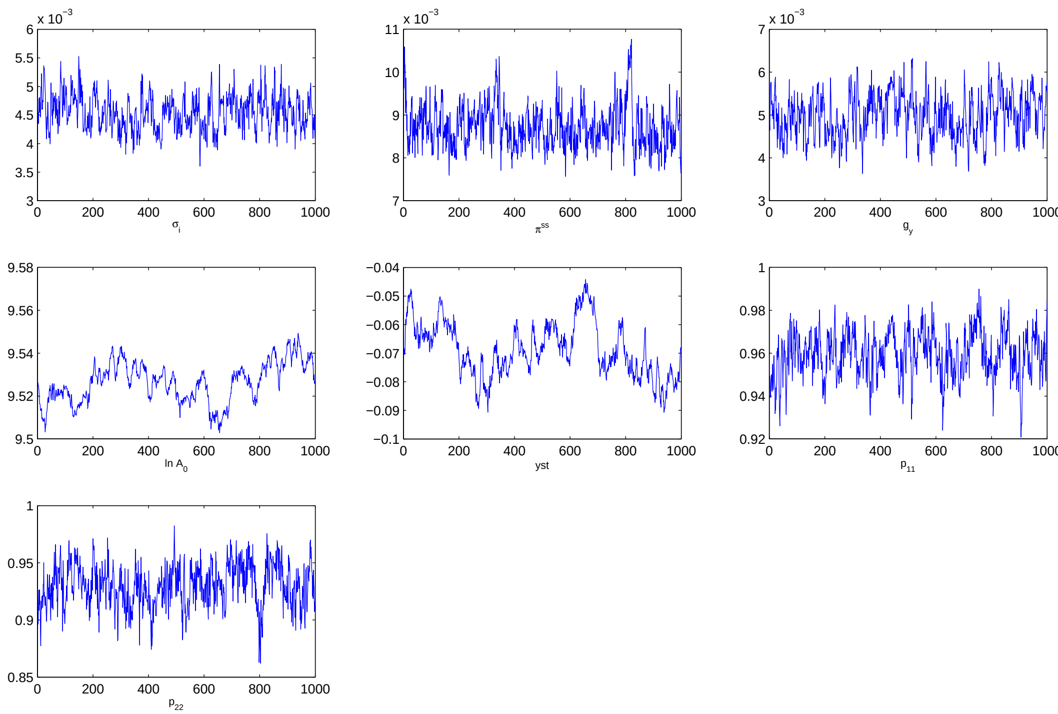

Figure 3: Trace Plots 1 : Switching Volatility

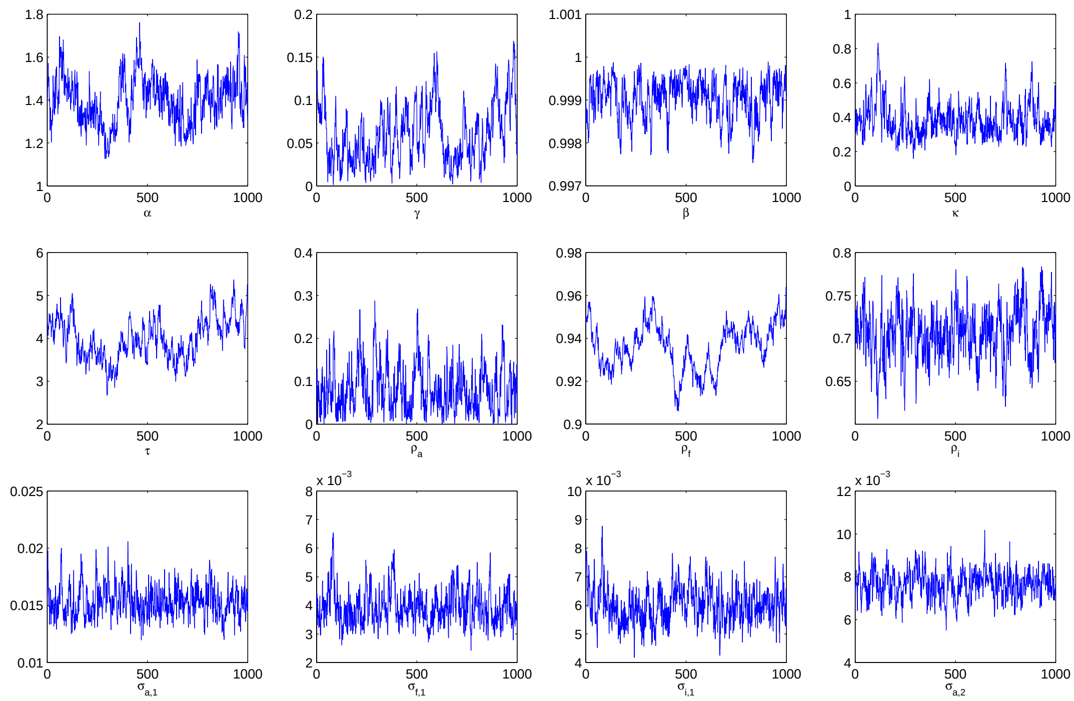

Figure 4: Trace Plots 2 : Switching Volatility

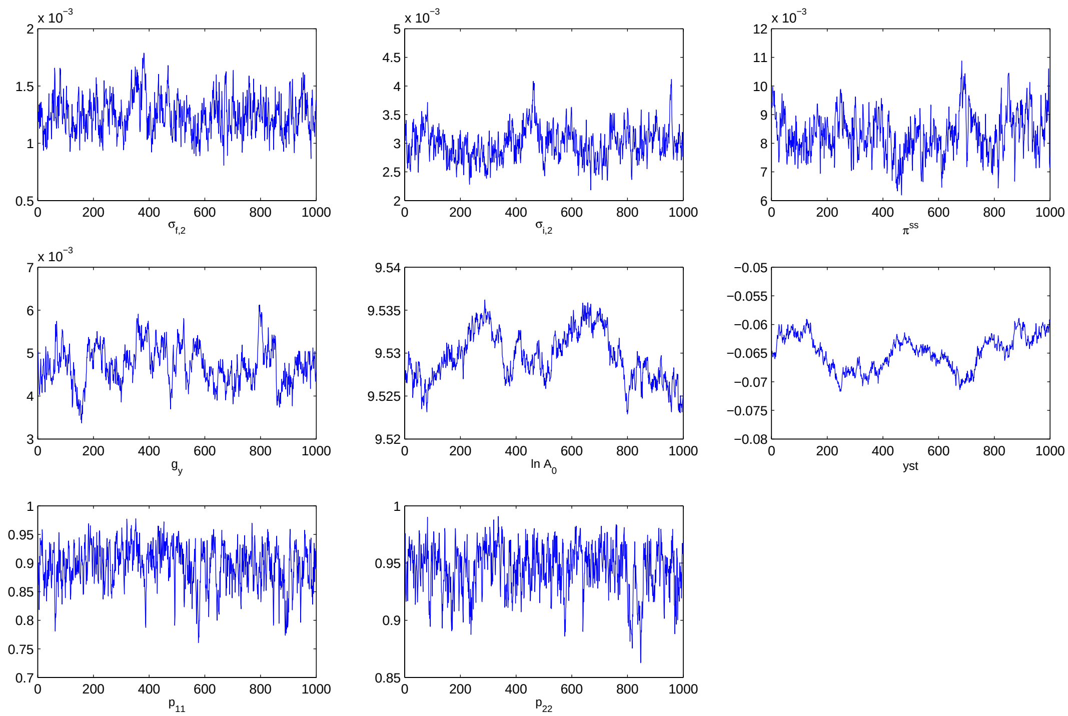

Figure 5: Trace Plots 1 : Switching Policy and Volatility

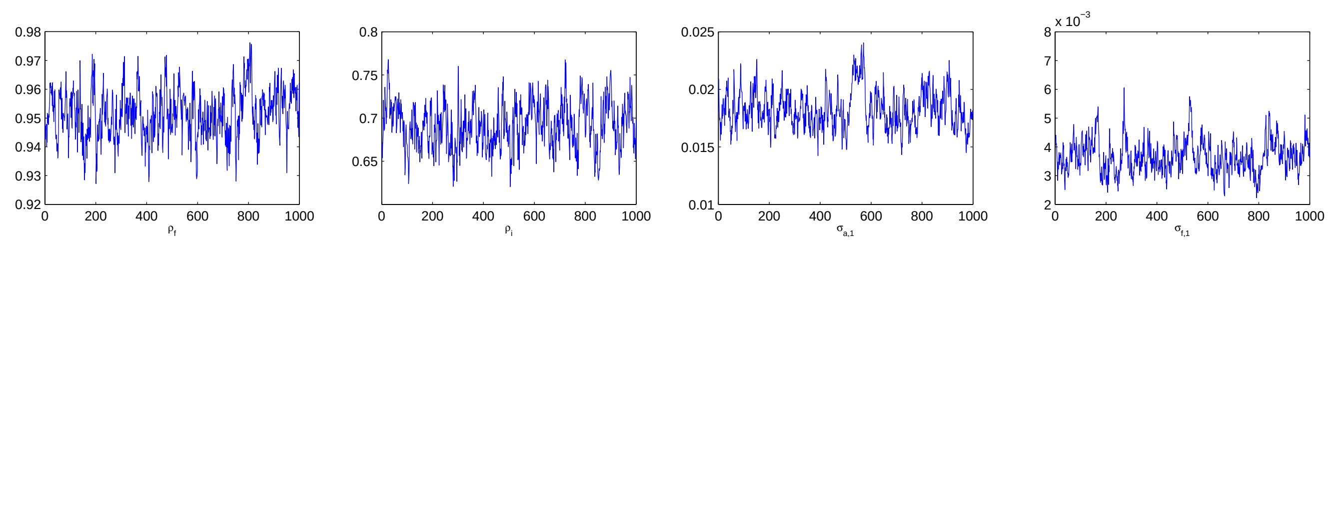

Figure 6: Trace Plots 2 : Switching Policy and Volatility

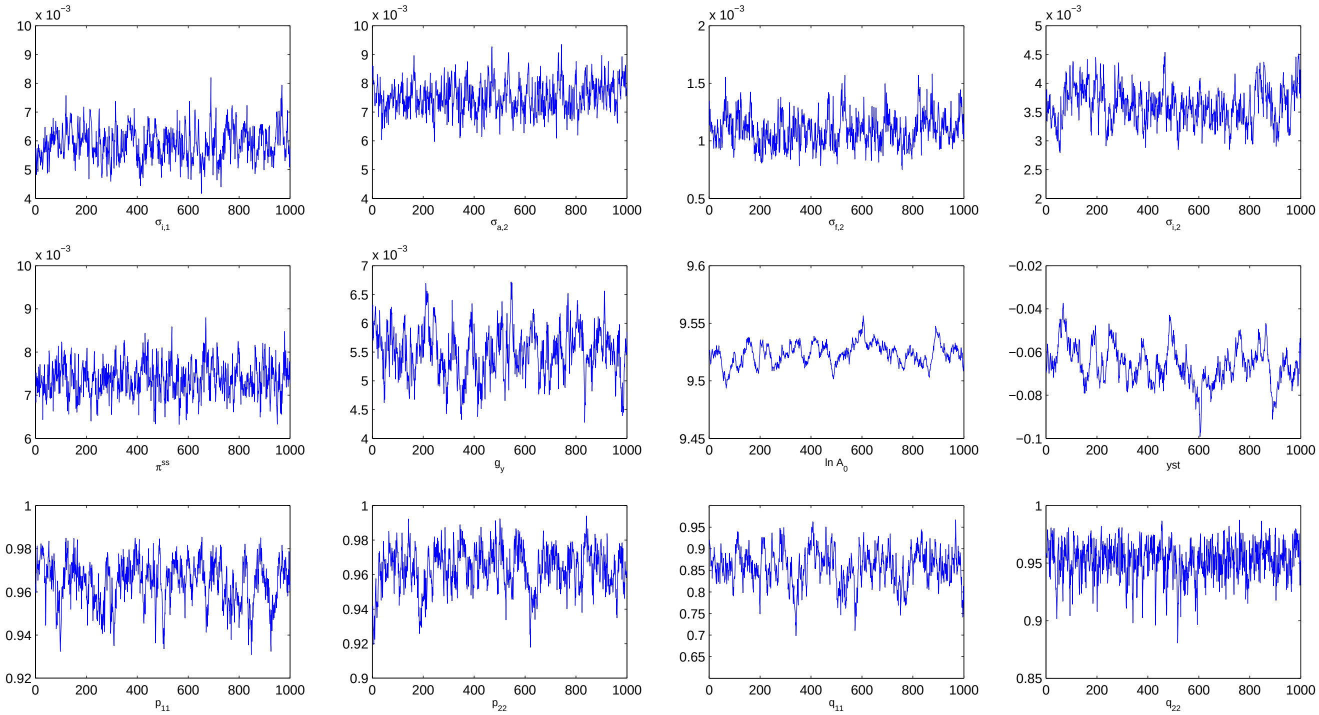

Figure 7: Prior-Posterior Moments of $AR(\pi)$: Switching Policy

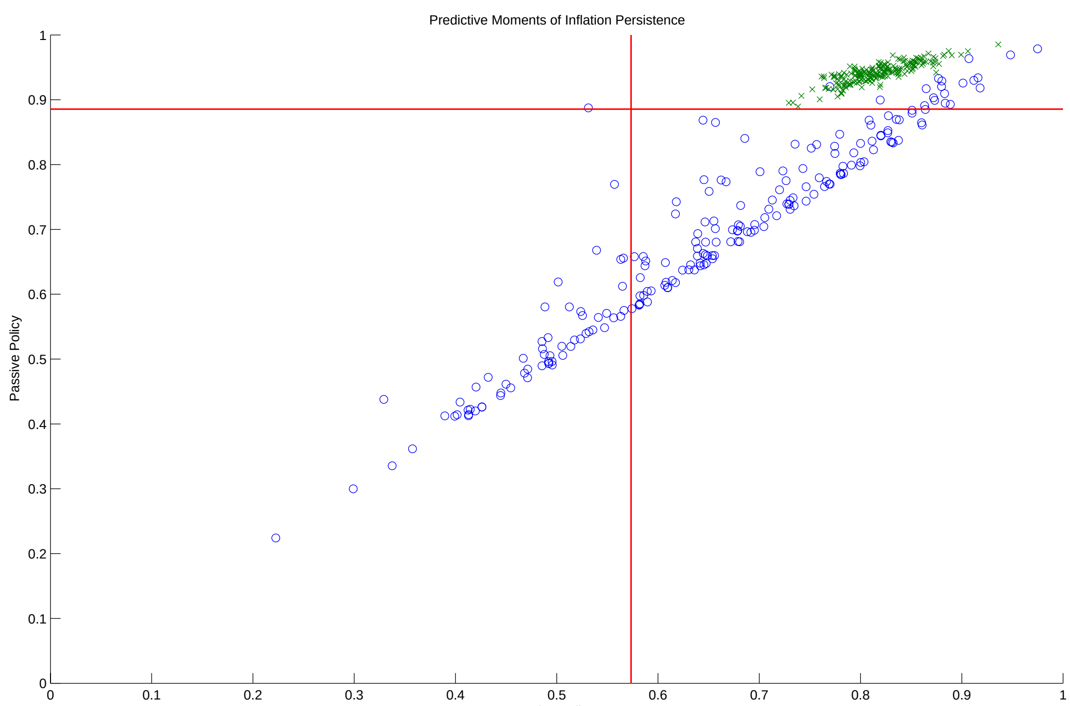

Figure 8: Prior-Posterior Moments of $AR(\pi)$: Switching Volatility

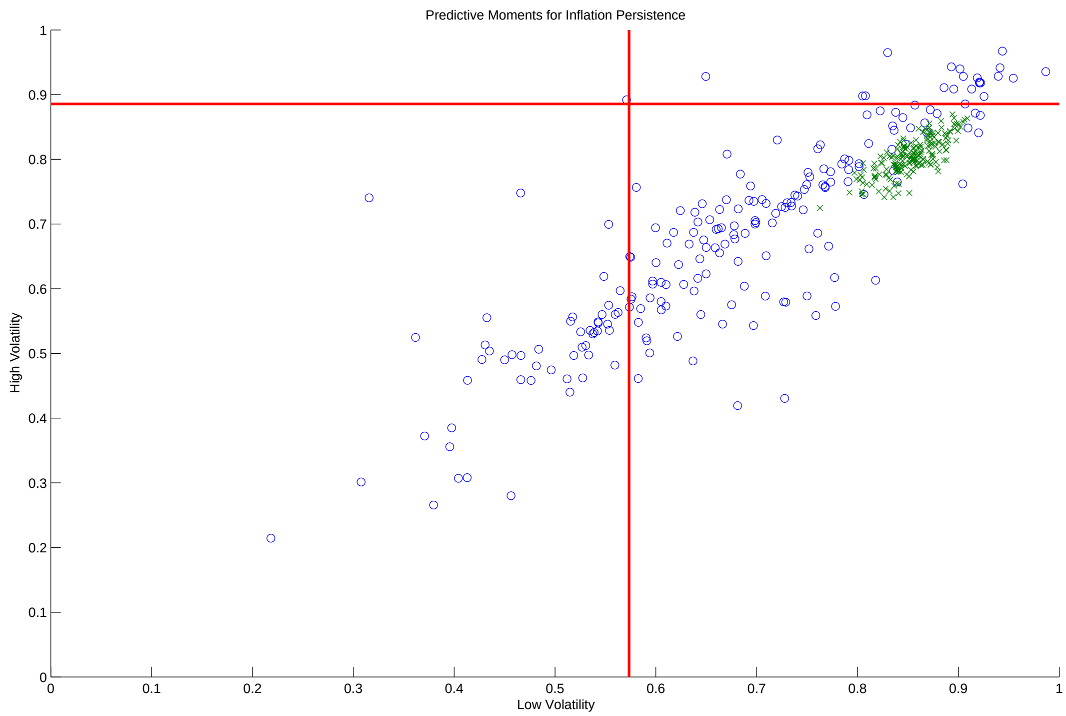

Figure 9: Prior-Posterior Moments of $AR(\pi)$: Switching Policy and Volatility