Microfoundations of Two-sided Markets: The Payment Card Example*

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Abstract

This paper provides a theory of two-sided market dynamics with arguably better microfoundations. These alternative microfoundations focus on observable heterogeneities of both sides of the market in a competitive framework. The theory is rich in empirical predictions and is less dependent on a particular form of imperfect competition than other approaches. Our findings in the payment card example point to adoption costs and the distribution of consumer incomes and firm sizes as the key determinants of the shares of costs borne by each side. This result provides clear implications for industry dynamics and sheds light on the puzzle of asymmetric pricing.

Keywords: Technology Adoption, Two-sided Market, Asymmetric Pricing

JEL Classification: L10, D40, O30

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1 Introduction

1.1 Motivation

The model of two-sided markets adds an important new chapter in the field of industrial organization. Researchers have applied the model to payment markets, media and advertising markets, dating clubs, game system markets, and many others. The common microfoundations of most of the models of two-sided markets include heterogeneity of preferences, imperfect competition, and adoption costs on only one side of the market. We offer an alternative, arguably better, set of microfoundations for the two-sided market model. We apply this approach to payment cards and illustrate the industry dynamics. This exercise offers many empirically testable hypotheses to explain the evolution of interchange fees in various countries.

Of most note, our motivation for the heterogeneity of demands on the two sides of the market focuses on empirically observable variables. This contrasts with the standard approach of assuming a heterogeneous distribution of unobservable "convenience benefits" from the use of a payment card for both sides of the market. In this regard, the standard approach can be thought of as a reduced form of a model such as ours. For example, in modelling the demand for payment cards we posit that consumers face both an adoption costs and a variable fee for use of a payment card. For high-income consumers, adopting the card can yield cost savings. As a result, demand for payment cards by consumers is heterogeneous because consumer incomes are heterogeneous. On the merchant side of the payment market we again posit that the merchant faces fixed adoption costs and variable fees for use. These conditions suggest that larger merchants will find adoption more economical than smaller merchants; the size distribution of firms, a distribution determined by technological considerations of costs of entry, economies of scale, and the preferences of consumers, is the empirical distribution that yields demand heterogeneity on this side of the market.

Our emphasis on heterogeneity in endowments and technology yields more empirically testable hypotheses than the assumption of unobservable heterogeneity in preferences. As we will discuss, the basic empirical predictions of our approach are broadly consistent with industry evidence, and would not be generated in the more standard approach without additional assumptions on the nature of preferences. Furthermore, the approach can be applied to other two-sided market contexts. Consider dating clubs, for example. Rather than to assume that men and women have systematically different preferences for dates or matches, our approach would be to assume common preferences for dating matches for both men and women. Heterogeneity of population, income, and physical features such as height, among men and women would then affect the determination of the prices for admission to the dating club in a particular market. We are confident that this approach is more fruitful empirically, requires fewer ancillary assumptions on the form of preferences, and is more economic in its approach, in which differing incentives, rather than differing preferences, explain heterogeneous behavior.

Another feature of our model that allows us to more easily investigate the dynamics of two-sided markets is the assumption of adoption costs on both sides of the market. This assumption yields an important benefit, namely, it allows us to use a contestable market structure among merchants. The benefits of employing a contestable model is that it is a more applicable model to many industries than any particular model of imperfect competition. In a particular application, a model of imperfect competition might be more apposite, but for a generalized purpose, the contestable market model is more parsimonious.

In the payment card context, it has previously been pointed out (Wright (2003)) that a merchant accepting both cards and cash and who is subject to competition from specialized merchants accepting only cash or cards, would be competed out of business. The contestable, or competitive market, structure that we employ allows for the entry, or the threat of entry, from specialized merchants–those who accept only cash, for example, or price their product higher than a cash-only merchant would to attract only card customers. However, we find that large merchants who accept both cash and payment cards do survive the threat of entry from specialized merchants. They survive the threat of entry because of the presence of adoption costs of cards. Because the adoption costs can be spread over a large number of transactions, and because the variable fees of card use are less than the variable costs of handling cash, the large merchants who adopt cards can offer lower prices than a cash-only specialized merchant.

Based on the contestability of the market and on payment card adoption costs, our equilibrium is then characterized by two thresholds in merchant size. Large merchants adopt payment cards and set a price that is lower than cash customers would experience at a cash-only merchants. So the large merchants attract customers who pay with either cash or the card. Medium size merchants, in contrast, are specialized. Some of these merchants accept only cash. The others adopt cards, but set a price that is higher than the competing cash-only merchants. They attract only consumers that use cards, because those consumers are better-off paying the higher price using their card, because cards are cost-saving to them overall. Finally, small merchants are all cash-only merchants. In other applications, such as game systems, for example, this pattern of equilibrium adoption and pricing by merchants would result in large game system writers offering games that can be used on alternate systems, medium size ones tending to specialize in one game system or another, and small firms writing games that can be used on the system with the lowest adoption costs.

Our assumption that yields two-sided markets is the assumption that merchants who adopt cards cannot price discriminate based on the consumer's choice of payment method–often called "price coherence." This assumption is a common one in the payment card literature. A restriction on price discrimination is used in many areas of economics, and is an empirically testable assumption.

Turning to our specific payment card example, this approach to modeling the adoption of payment devices results in a model of a two-sided market. However, it stands in contrast to much of the literature regarding payment devices, with the notable exceptions of Farrell (2006) and Rochet and Tirole (2006). In the more standard approach to modeling payment device markets in the two-sided market literature, as in Baxter (1983), Rochet and Tirole (2000), Schmalensee (2002), and Wright (2003), consumers and merchants derive benefits b_c and b_m, respectively, from their use of a particular payment device. Under conditions that lead to two-sided markets (Rochet and Tirole (1999)) interchange fees then play a role in balancing the demands on the two sides of the markets for some objective, either to maximize transaction volume (welfare) or to maximize the profits of the provider of the payment device. This modeling technique has the merchants and consumers in essentially symmetric positions, both having direct demands for the payment device.

In the now standard approach, the consumer's benefit from the use of the card, b_c, is referred to as the convenience benefit from the payment device. In contrast, our approach looks at consumers as deriving utility from their consumption of goods which they purchase subject to an income constraint. The payment method the consumer uses does not yield utility directly, but instead imposes a frictional cost on their purchases. The standard models are only partially applicable to payment devices, and only so when the payment device, such as a payment card, offers some direct benefit (utility) to the consumer in addition to the ability to make monetary

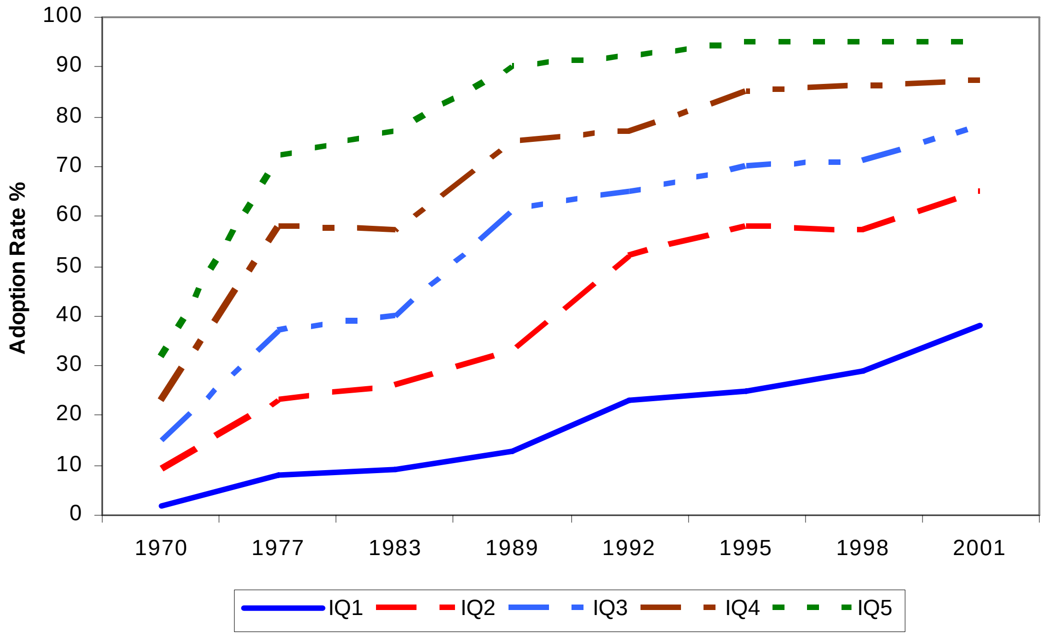

Figure 1: Household Credit Card Adoption by Income Quintile

transfers. In contrast, the model we explore ignores any nonmonetary characteristic of the payment device, and examines adoption and use in the environment in which the monetary characteristics of payment devices are their only raison d'être.

This approach yields clear empirically relevant hypotheses. For consumers, consider the introduction of a payment device with a high fixed but low variable cost of use. More affluent consumers, with higher levels of consumption and purchases, will choose to adopt the device prior to less affluent consumers. For merchants, facing a similar adoption decision, the larger merchants, or those who sell a higher valued good, will adopt the device earlier than other merchants. These predictions are consistent with empirical evidence (Figure 1 and 2)[1]. In contrast, the literature that overlooks the monetary nature of payment devices does not yield such straightfor-

[1]Data source: Evans and Schmalensee (2005), *Paying with Plastic*, 2nd edition.

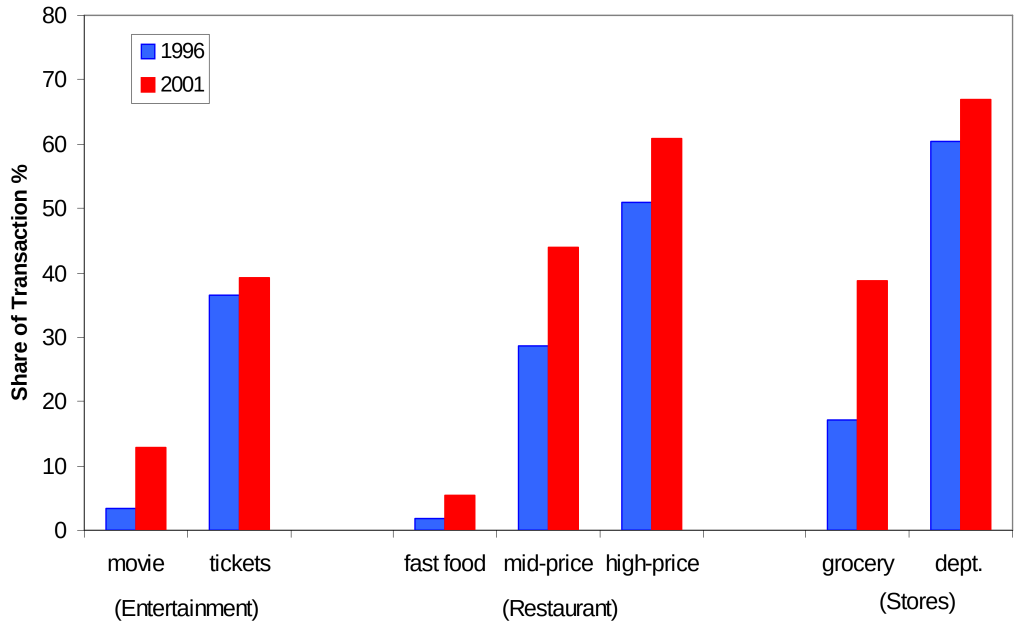

Figure 2: Payment Card Share of Transaction Volume by Merchant Type

ward empirical conclusions without additional assumptions about how the specific convenience benefits are distributed among consumers and merchants.

By focusing on the moneyness of payment devices, we might be criticized for overlooking nonmonetary benefits consumers or merchants might derive from their use. We offer three defences. First, the monetary nature of payment devices is arguably their primary purpose. Second, many convenience benefits of payment devices (e.g. protect from theft or time saving), are closely related with the income and spending of the consumer, and are therefore better captured by our model through the variable cost of use of the payment device. Third, it may be appropriate to model both the monetary and other, direct, benefits of a payment device. But we believe that only by first investigating the adoption pattern of a monetary payment device can we understand the circumstances under which sellers of payment devices will choose to employ a strategy of tying a direct benefit (not related to the income of the consumer) to the use of the device, and determining on which side of the market those benefits might be offered. By overlooking the monetary nature of payment devices, one is apt to misunderstand the basic asymmetry between the economic roles of the consumer and the merchant.

1.2 A New Approach

We model the consumers as having generalized Cobb-Douglas preferences across a range of goods. They take prices as given. Each consumer is endowed with income, which is distributed across the population of consumers according to known cumulative distribution function. The merchant side of our model is quite stylized. Each merchant competes in a contestable market for the single good the merchant sells, and prices are set at the zero profit level. The size of an individual merchant is hence tied to the consumers' demand.

Consumers and merchants are both presented with the option to adopt a new payment device that offers a lower variable cost of use, but a higher fixed cost relative to the pre-existing alternative. They each make their optimal adoption decision taking the other's choice as given. The model yields a two-sided market, given the heterogeneity of consumer incomes and merchant sizes and under price coherence of merchants that accept both payment devices. We then examine the adoption decisions under various market structures for the provision of the payment device, including a competitive (zero-profit) market structure in which no interchange fees are feasible, a competitive (or zero-profit) structure in which interchange fees are feasible, a monopoly structure, and the solution that would be determined by a Ramsey social planner. Our analyses show that consumer income, adoption cost, and market structure each play important roles in determining the pricing and usage of payment devices. Moreover, we find that no market structure yields the planner's solution, in contrast with some previous literature, including Schmalensee (2002).

Our model can be readily applied to the payment card industry. It suggests that both the increasing concentration of payment card networks and the growth of consumer incomes relative to card service costs may help explain the puzzles surrounding interchange fees pointed out in Hayashi (2005) and Weiner and Wright (2005). Here, it is worth emphasizing the differences between our model and others. The existing studies on payment card market typically assume imperfect competition among merchants, e.g. Hotelling competition. Those models allow the merchants to behave strategically and consider the business stealing motive for adopting payment cards, but can not easily keep track of industry dynamics. In contrast, our model assumes competitive merchants and highlights the positive and normative consequences of market structure, income growth, and adoption costs in a nonstrategic (merchant) environment. The richness of the strategic approach is sacrificed in favor of a focus on the interplay between individual firm and consumer decision-making and aggregate industry characteristics. As a result, our model provides a convenient framework to study evolution of payment card industry both in the short run (illustrating the network "chicken-egg" dynamics) and long-run (illustrating adoption and pricing dynamics due to cost and income changes), and offers some further insights into the related competitive policy issues.

The modelling approach we've laid out in this paper is applicable to other industries. By focusing on distributions of endowments and firm size and by placing the economic interaction among agents in the model in a contestable environment, this approach is potentially more fruitful in empirical measurement and hypothesis testing.

1.3 Road Map

In the next section we lay out our model in greater detail and derive some preliminary results. In section 3 we review numerical analyses of the equilibria of our model, and apply our findings to the interchange fee puzzles of payment cards. In section 4 we offer concluding remarks and suggestions for future research.

2 The Model

Here we present our model to study pricing and adoption of monetary payment devices. We first lay out the environment in which only one payment device, which we refer to as cash, is in use. Later we will consider the introduction of an alternative device, which we refer to as a payment card.

2.1 Pre-card Market Environment

The economy is composed of a continuum of merchants. Each merchant locates in a physical store and sells a distinct product α. The store facility is sunk and each product market is contestable, so merchants always sell at cost:

$$(1-\tau_m)p_\alpha = c_\alpha \Longrightarrow p_\alpha = \frac{c_\alpha}{1-\tau_m}$$

where p_α and c_α are price and cost for good α respectively; τ_m is the cash payment cost to the merchant. The cost of the cash payment includes the handling, storage, and safekeeping costs the merchant expends in accepting cash.

A consumer has generalized Cobb-Douglas preference. She would like to consume all varieties of products and seeks to maximize her utility subject to her income I:

$$U = Max \int_{\underline{\alpha}}^{\overline{\alpha}} \alpha \ln x_\alpha dG(\alpha) \qquad s.t. \qquad \int_{\underline{\alpha}}^{\overline{\alpha}} (1+\tau_c)p_\alpha x_{\alpha,I} dG(\alpha) = I$$

where $\alpha \in [\underline{\alpha}, \overline{\alpha}]$ is the preference parameter distributed with cdf $G(\alpha)$, $x_{\alpha,I}$ is her quantity of demand for good α, τ_c is the cost of a cash payment to the consumer. As with the merchant, the consumer faces costs in handling and transporting cash.

Therefore, the demand and spending of an individual consumer on good α can be determined as

$$x_{\alpha,I} = \frac{\alpha I}{(1+\tau_c)p_\alpha E(\alpha)}; \quad p_\alpha x_{\alpha,I} = \frac{\alpha I}{(1+\tau_c)E(\alpha)}$$

Across consumers, the income $I \in [\underline{I}, \overline{I}]$ is distributed with cdf function $F(I)$ and mean $E(I)$. Normalize the aggregate measure of consumer to be unity. At equilibrium, market supply equals demand, so the market output and value for product α are as follows:

$$x_\alpha = \frac{\alpha E(I)}{(1+\tau_c)p_\alpha E(\alpha)}; \quad p_\alpha x_\alpha = \frac{\alpha E(I)}{(1+\tau_c)E(\alpha)}$$

2.2 Card Adoption and Market Equilibrium

At time T, a payment innovation, e.g., a card, is introduced. The card service is provided by a card network, who charges merchants and consumers a proportional fee f_m and f_c respectively. The costs of providing the card service to merchants and consumers are d_m and d_c, respectively. For merchants and consumers, there is a per-period adoption cost k_m (e.g., a fixed cost of renting card-processing equipment) and k_c (e.g., a fixed cost of maintaining banking account balance or credit score). At equilibrium, large merchants and wealthy consumers have an advantage in adopting the payment card. To see that, let us construct the following equilibrium: given that merchants $\alpha \geq \alpha_0$ accept the card, consumers of income $I \geq I_0$ would like to adopt the card, and vice versa.

2.2.1 Merchants' Choice

Merchants take consumers' card adoption as given to make their card acceptance decision. Due to competition in the contestable market, they fall into three categories based on their transaction volume: (1) Large merchants ($\alpha \geq \alpha_1$) accept card and charge price $p_{\alpha,d} \leq p_{\alpha,c}$ so that they are patronized by both card and cash customers; (2) Intermediate merchants ($\alpha_0 \leq \alpha < \alpha_1$) specialize. They either accept card and charge $p_{\alpha,d}$, where $\frac{1+\tau_c}{1+f_c}p_{\alpha,c} \geq p_{\alpha,d} > p_{\alpha,c}$, so that they are patronized only by card customers, or they do not accept card and charge $p_{\alpha,c}$ so that they only serve cash customers. (3) Small merchants ($\alpha < \alpha_0$) do not accept card and charge $p_{\alpha,c}$, so that all customers shop there with cash.

Category (1): $\alpha \geq \alpha_1$ To elaborate on this, let's start with the first category. Merchants in this category receive revenue from card and cash customers, respectively

$$p_{\alpha,d}x^{card}_{\alpha,d} = \frac{\alpha[E_{I>I_0}(I-k_c)]}{E(\alpha)(1+f_c)}; \quad p_{\alpha,d}x^{cash}_{\alpha,d} = \frac{\alpha[E_{I<I_0}(I)]}{E(\alpha)(1+\tau_c)} \tag{1}$$

where $E_{I>I_0}(I) \equiv \int_{I_0}^{\overline{I}} IdF(I)$.

Contestability requires zero profit so that revenue equals cost,

$$(1-f_m)p_{\alpha,d}x^{card}_{\alpha,d} + (1-\tau_m)p_{\alpha,d}x^{cash}_{\alpha,d} = c_\alpha x^{card}_{\alpha,d} + c_\alpha x^{cash}_{\alpha,d} + k_m \tag{2}$$

Equation 1 and 2 pin down the price $p_{\alpha,d}$:

$$p_{\alpha,d} = \frac{c_\alpha \frac{\alpha[E_{I>I_0}(I-k_c)]}{(1+f_c)} + c_\alpha \frac{\alpha[E_{I<I_0}(I)]}{(1+\tau_c)}}{(1-f_m)\frac{\alpha[E_{I>I_0}(I-k_c)]}{1+f_c} + (1-\tau_m)\frac{\alpha[E_{I<I_0}(I)]}{1+\tau_c} - k_m E(\alpha)}$$

Given $p_{\alpha,d} \leq p_{\alpha,c} = \frac{c_\alpha}{1-\tau_m}$, a merchant has to be large enough to be in this category, i.e., $\alpha \geq \alpha_1$, where

$$\alpha_1 = \frac{E(\alpha)k_m}{[E_{I>I_0}(I-k_c)](\frac{1-f_m}{1+f_c} - \frac{1-\tau_m}{1+f_c})} \tag{3}$$

Category (2): $\alpha_0 \leq \alpha < \alpha_1$ Merchants in this intermediate group specialize. For each product, there are two merchants. One accepts card and charges $p_{\alpha,d}$, where $\frac{1+\tau_c}{1+f_c}p_{\alpha,c} \geq p_{\alpha,d} > p_{\alpha,c}$, so that it is patronized only by card customers. The other does not accept card and charge $p_{\alpha,c}$ so that it only serves cash customers.

A card merchant receives revenue only from card customers and earn zero profit, which implies

$$p_{\alpha,d} = \frac{c_\alpha \frac{\alpha[E_{I>I_0}(I-k_c)]}{(1+f_c)}}{(1-f_m)\frac{\alpha[E_{I>I_0}(I-k_c)]}{1+f_c} - k_m E(\alpha)}$$

Therefore, $\frac{1+\tau_c}{1+f_c}p_{\alpha,c} \geq p_{\alpha,d} > p_{\alpha,c}$ implies that merchants with $\alpha_0 \leq \alpha < \alpha_1$ are in this intermediate group, where α_1 is given in Equation 3 and

$$\alpha_0 = \frac{E(\alpha)k_m}{[E_{I>I_0}(I-k_c)](\frac{1-f_m}{1+f_c} - \frac{1-\tau_m}{1+\tau_c})}$$

Category (3): $\alpha < \alpha_0$ Small merchants with $\alpha < \alpha_0$ are in the third category. Given their small transaction volumes, accepting card will result $p_{\alpha,d} > \frac{1+\tau_c}{1+f_c}p_{\alpha,c}$. Therefore, all customers shop there with cash.

This arrangement of prices and shopping patterns suggests that different economics are at play than in other models of card adoption, for example Rochet and Tirole (2006). In this equilibrium we'll find that for every card-adopting merchant, the merchant prefers the use of the card by the consumer to the use of cash, both on an ex ante and ex post basis. Merchants who find card transactions more costly than cash transactions either charge higher prices that compensate for the higher cost or decline to accept cards.

2.2.2 Consumers' Choice

An individual consumer takes market prices and merchants' card acceptance as given to make her own adoption decision. Given that merchants $\alpha \geq \alpha_0$ accept the card, she compares the utility of adopting card or not. An adopter who enjoys higher utility from adoption meets the following condition:

$$\int_{\underline{\alpha}}^{\alpha_1} \alpha \ln \frac{\alpha I}{(1+\tau_c)p_{\alpha,c}E(\alpha)} dG(\alpha) + \int_{\alpha_1}^{\overline{\alpha}} \alpha \ln \frac{\alpha I}{(1+\tau_c)p_{\alpha,d}E(\alpha)} dG(\alpha)$$
$$\leq \int_{\underline{\alpha}}^{\alpha_0} \alpha \ln \frac{\alpha (I-k_c)}{(1+\tau_c)p_{\alpha,c}E(\alpha)} dG(\alpha) + \int_{\alpha_0}^{\overline{\alpha}} \alpha \ln \frac{\alpha (I-k_c)}{(1+f_c)p_{\alpha,d}E(\alpha)} dG(\alpha)$$

which implies that

$$E(\alpha) \ln(\frac{I}{I-k_c}) + \int_{\alpha_0}^{\alpha_1} \alpha \ln(\frac{p_{\alpha,d}}{p_{\alpha,c}}) dG(\alpha) < E_{\alpha>\alpha_0}(\alpha) \ln(\frac{1+\tau_c}{1+f_c}) \tag{4}$$

where $E_{\alpha>\alpha_0}(\alpha) \equiv \int_{\alpha_0}^{\overline{\alpha}} \alpha dG(\alpha)$.

Equation 4 suggests that for any individual consumer to adopt card, we need

$$\tau_c > f_c$$

and the adopters' income has to be over the threshold level I_0

$$I \geq I_0 = \frac{(\frac{1+\tau_c}{1+f_c})^{E_{\alpha>\alpha_0}(\alpha)/E(\alpha)} k_c}{(\frac{1+\tau_c}{1+f_c})^{E_{\alpha>\alpha_0}(\alpha)/E(\alpha)} - \exp(\int_{\alpha_0}^{\alpha_1} \alpha \ln(\frac{p_{\alpha,d}}{p_{\alpha,c}}) dG(\alpha)/E(\alpha))}$$

2.2.3 Market Equilibrium

As discussed, the interrelationship between consumers' card adoption and merchants' card acceptance can be summarized as follows:

$$\alpha_1 = \frac{E(\alpha)k_m}{[E_{I>I_0}(I-k_c)](\frac{1-f_m}{1+f_c} - \frac{1-\tau_m}{1+f_c})} \tag{5}$$

$$\alpha_0 = \frac{E(\alpha)k_m}{[E_{I>I_0}(I-k_c)](\frac{1-f_m}{1+f_c} - \frac{1-\tau_m}{1+\tau_c})} \tag{6}$$

$$\frac{p_{\alpha,d}}{p_{\alpha,c}} = \frac{\alpha[E_{I>I_0}(I-k_c)](1-\tau_m)}{(1-f_m)\alpha[E_{I>I_0}(I-k_c)] - k_m E(\alpha)(1+f_c)} \tag{7}$$

$$I_0 = \frac{(\frac{1+\tau_c}{1+f_c})^{E_{\alpha>\alpha_0}(\alpha)/E(\alpha)} k_c}{(\frac{1+\tau_c}{1+f_c})^{E_{\alpha>\alpha_0}(\alpha)/E(\alpha)} - \exp(\int_{\alpha_0}^{\alpha_1} \alpha \ln(\frac{p_{\alpha,d}}{p_{\alpha,c}}) dG(\alpha)/E(\alpha))} \tag{8}$$

To simplify the analysis, let us introduce some notations here:

$$Z_1 = (\frac{1-f_m}{1+f_c} - \frac{1-\tau_m}{1+f_c}); \quad Z_0 = (\frac{1-f_m}{1+f_c} - \frac{1-\tau_m}{1+\tau_c})$$

Therefore, Equations 5, 6 and 7 imply

$$\alpha_1 = \frac{Z_0}{Z_1}\alpha_0; \quad \frac{p_{\alpha,d}}{p_{\alpha,c}} = \frac{(1-\tau_m)\alpha}{(1-f_m)\alpha - (1+f_c)\alpha_0 Z_0}$$

As a result, the equilibrium conditions 5 - 8 can be rewritten as follows:

$$\alpha_0 = \frac{E(\alpha)k_m}{[E_{I>I_0}(I-k_c)]Z_0} \tag{9}$$

$$I_0 = \frac{(\frac{1+\tau_c}{1+f_c})^{E_{\alpha>\alpha_0}(\alpha)/E(\alpha)} k_c}{(\frac{1+\tau_c}{1+f_c})^{E_{\alpha>\alpha_0}(\alpha)/E(\alpha)} - \exp(\int_{\alpha_0}^{\frac{Z_0}{Z_1}\alpha_0} \alpha \ln(\frac{(1-\tau_m)\alpha}{(1-f_m)\alpha-(1+f_c)\alpha_0 Z_0}) dG(\alpha)/E(\alpha))} \tag{10}$$

We are now ready to discuss equilibrium outcomes under four alternative payment market structures as follows.

(1) Competitive Network without Interchange Fee First, in a competitive card service market where it is not feasible to assess an interchange fee, we have

$$f_m = d_m \quad \text{and} \quad f_c = d_c$$

and the adoption thresholds are given by Equations 9-10. The corresponding card transaction volume is

$$\frac{E_{\alpha>\alpha_0}(\alpha)E_{I>I_0}(I-k_c)}{E(\alpha)(1+d_c)}$$

(2) Competitive Network with Interchange Fee If charging an interchange fee is feasible, a competitive card network (e.g., a non-profit bank association) can achieve more card transactions. The interchange fee would be defined as the transfer from merchants to consumers in the amount $f_m - d_m = -(f_c - d_c)$ The network will have the following objective and constraints in setting the merchant and consumer fees (the price structure):

$$\underset{f_c, f_m}{Max} \frac{E_{\alpha > \alpha_0}(\alpha) E_{I > I_0}(I - k_c)}{E(\alpha)(1 + f_c)}$$

$$s.t. \quad Equations\ 9 - 10; \qquad f_m + f_c = d_c + d_m$$

(3) Monopoly Network A monopoly card network would like to maximize the card revenue instead of transaction volume. It solves the following problem:

$$\underset{f_c, f_m}{Max} \frac{E_{\alpha > \alpha_0}(\alpha) E_{I > I_0}(I - k_c)}{E(\alpha)(1 + f_c)} (f_c + f_m - d_m - d_c)$$

$$s.t. \quad Equations\ 9 - 10$$

(4) Social Planner The social planner would like to maximize the social surplus of using a more efficient payment device, taking into account the adoption costs of consumers and merchants, and subject to the incentive constraints of both. In addition, the social planner also faces the balanced-budget constraint (Ramsey pricing).

$$\underset{f_c, f_m}{Max} \quad \frac{E_{\alpha > \alpha_0}(\alpha) E_{I > I_0}(I)}{E(\alpha)(1 + \tau_c)} (\tau_c + \tau_m) - \frac{E_{\alpha > \alpha_0}(\alpha) E_{I > I_0}(I - k_c)}{E(\alpha)(1 + f_c)} (d_m + d_c)$$
$$+ \frac{E_{\alpha < \alpha_0}(\alpha)(E_{I > I_0}(I) - E_{I > I_0}(I - k_c))}{E(\alpha)(1 + \tau_c)} (\tau_m + \tau_c) - (1 - G(\alpha_0)) k_m - (1 - F(I_0)) k_c$$

$$s.t. \quad Equations\ 9-10; \qquad f_m + f_c \geq d_c + d_m$$

3 Numerical Analysis

To better illustrate our findings, we consider an explicit example as follows.

Assume $\alpha \in (0,1)$ is uniformly distributed[2] with $E(\alpha) = 1/2$, and $I \in [0, \infty)$ is exponentially distributed with $F(I) = 1 - e^{(-\lambda I)}$ and $E(I) = 1/\lambda$. Note that $E_{\alpha > \alpha_0}(\alpha) = \frac{1-\alpha_0^2}{2}$ and $E_{I > I_0}(I - k_c) = e^{-\lambda I_0}(\frac{1}{\lambda} + I_0 - k_c)$. Therefore, Equations 9-10 can be rewritten into

$$\alpha_0 = \frac{k_m}{2e^{(-\lambda I_0)}(\frac{1}{\lambda} + I_0 - k_c)Z_0} \tag{L1}$$

$$I_0 = \frac{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2} k_c}{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2} - \exp(S\alpha_0^2)} \tag{L2}$$

where $S = \ln\frac{1+f_c}{1+\tau_c} + \frac{Z_0(1+f_c)}{(1-f_m)}(\frac{Z_0}{Z_1} - 1) + \frac{Z_0^2(1+f_c)^2}{(1-f_m)^2}\ln(\frac{\frac{Z_0}{Z_1} - \frac{Z_0(1+f_c)}{(1-f_m)}}{1 - \frac{Z_0(1+f_c)}{(1-f_m)}})$. A detailed proof of L2 is provided in the Appendix.

3.1 Short-run (Transitional) Dynamics

Characterizing Equation L1, we have

$$\alpha_0|_{I_0 \longrightarrow 0} \longrightarrow \frac{k_m}{2(\frac{1}{\lambda} - k_c)Z_0} > 0\ ; \quad \alpha_0|_{I_0 \rightarrow \infty} \longrightarrow \infty$$

$$\frac{d\alpha_0}{dI_0} > 0\ ; \quad \frac{d^2\alpha_0}{dI_0^2} > 0;$$

[2]The numerical analysis is not sensitive to the distribution of firm size. In the Appendix, we show that assuming α is exponentially distributed, i.e., $F(\alpha) = 1 - e^{(-\theta\alpha)}$, deliver similar results.

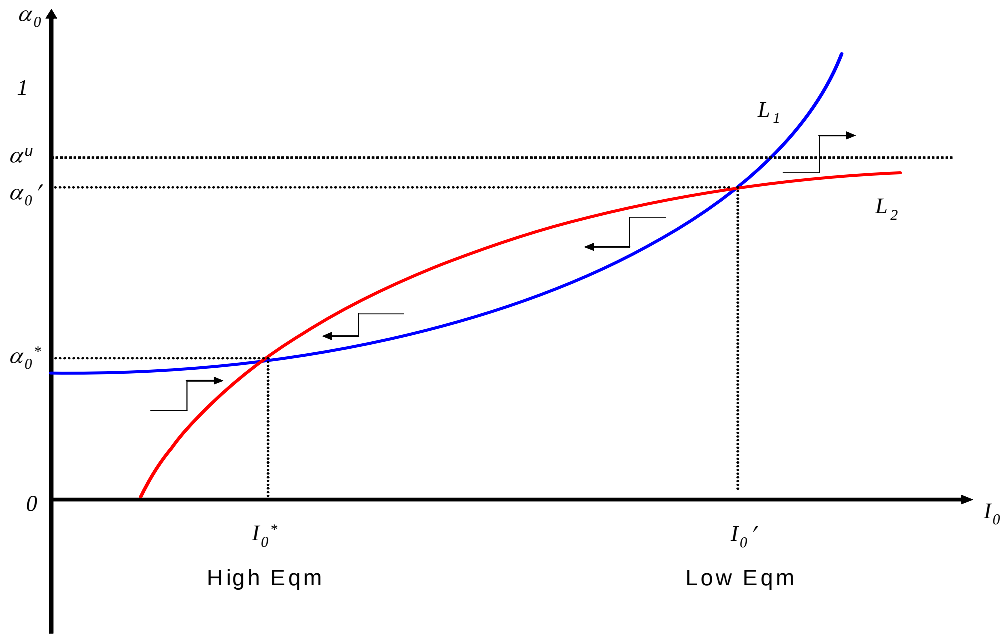

Figure 3: Interaction of Merchants and Consumers in Card Adoption

Characterizing Equation L2, we have

$$I_0|_{\alpha_0 \to 0} \to \frac{(\frac{1+\tau_c}{1+f_c})k_c}{(\frac{1+\tau_c}{1+f_c})-1} > 0 \; ; \quad \alpha_0|_{I_0 \longrightarrow \infty} \longrightarrow \alpha^u = (\frac{\ln(\frac{1+\tau_c}{1+f_c})}{\ln(\frac{1+\tau_c}{1+f_c})+S})^{1/2} < 1$$

$$\frac{d\alpha_0}{dI_0} > 0 \; ; \quad \frac{d^2\alpha_0}{dI_0^2} < 0$$

Figure 3 illustrates the interactions of card adoption between merchants and consumers and the corresponding transitional dynamics. There exist two steady states with positive levels of adoption (the no adoption outcome is a steady state as well): a high-adoption equilibrium (I_0^*,α_0^*) and a low-adoption equilibrium ($I_0^{'}$,$\alpha_0^{'}$). The high equilibrium is stable but the low equilibrium is not. As a result, the card network has incentive to push the card adoption to overcome the low equilibrium. Our analysis suggests if the initial card adoption is high enough, the market will achieve the high equilibrium. Otherwise, card adoption may fail, and suffer no adoption.

3.2 Long-run Dynamics

Using the high-adoption equilibrium, we can numerically compare the long-run industry dynamics under four different market structures. For the benchmark simulation, we use the following parameterization: $\tau_m = 0.05$, $\tau_c = 0.05$, $d_c < 0.05$, $d_m < 0.05, k_c = 125$, $k_m = 125$, $\lambda = 0.0001$. Based on that, we plot Figures 4 - 7 corresponding to each market structure. To study the comparative dynamics, we then adjust the values of k_c, k_m and λ to see the effects of changing consumer income and adoption costs on card pricing and usage (results are shown in the Appendix). In addition, we examine the outcome under an exponential distribution of firm size. That allows to examine the outcome under identical distributions of income and firm size.

3.2.1 Competitive Card Network without Interchange Fee

If it is not feasible for the competitive card network to set an interchange fee (other than zero), merchants and consumers will face their respective card service costs:

$$f_m = d_m \quad \text{and} \quad f_c = d_c$$

Therefore, the adoption thresholds are

$$\alpha_0 = \frac{k_m}{2e^{(-\lambda I_0)}(\frac{1}{\lambda} + I_0 - k_c)Z_0}; \quad I_0 = \frac{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2}k_c}{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2} - \exp(S\alpha_0^2)}$$

and the corresponding card transaction volume is

$$\frac{e^{(-\lambda I_0)}(\frac{1}{\lambda} + I_0 - k_c)(1 - \alpha_0^2)}{(1 + d_c)}$$

3.2.2 Competitive Card Network with Interchange Fee

If the competitive card network can set an interchange fee, then only the sum of card service costs $d_m + d_c$ matters. Therefore, the card network achieves better cost

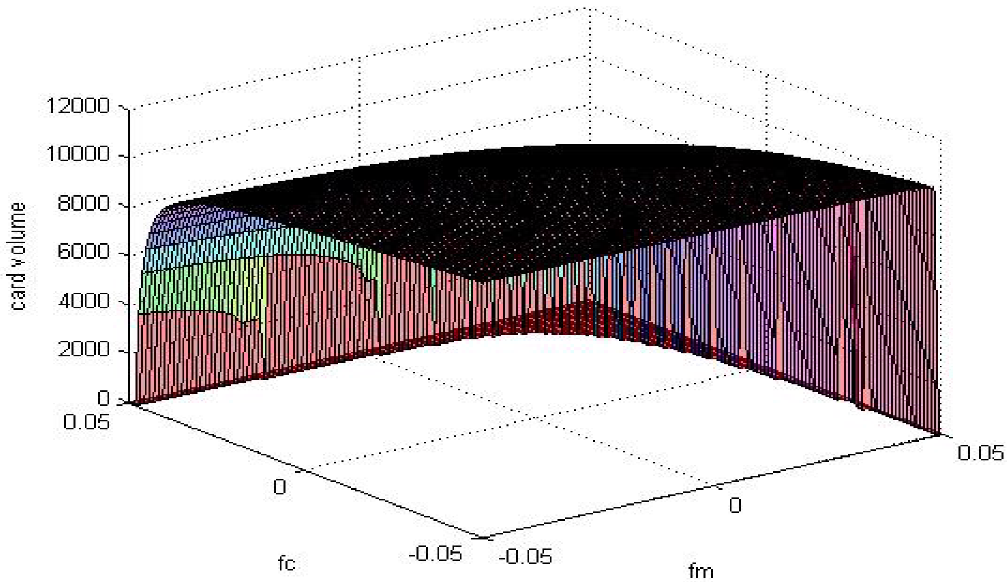

Figure 4: Card Fees and Transaction Volume

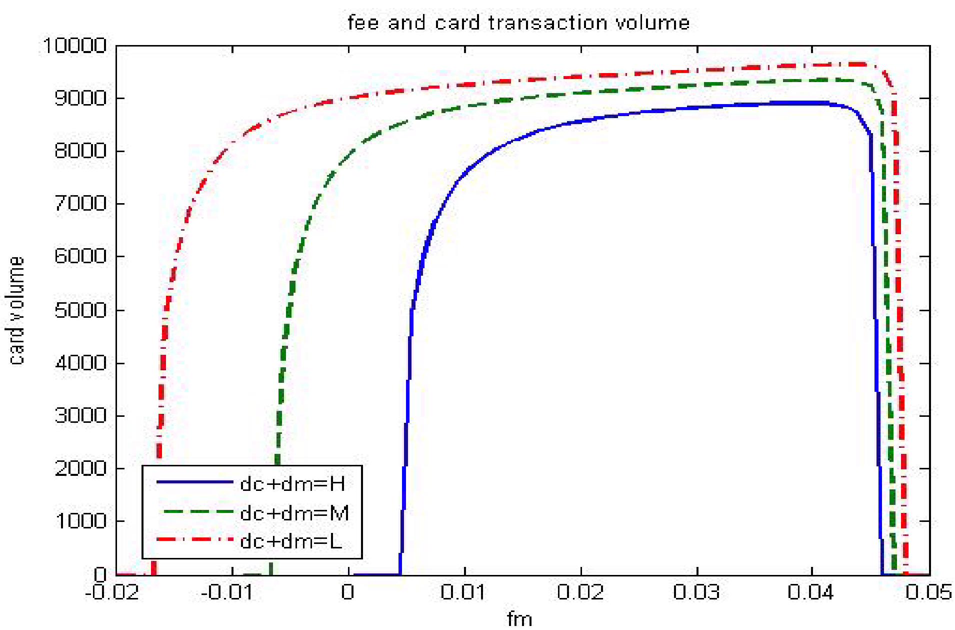

Figure 5: Cost Allocation and Card Transaction Volume

allocation and higher card adoption and usage.

$$\underset{f_c,f_m}{Max}\ \ e^{(-\lambda I_0)}(\frac{1}{\lambda}+I_0-k_c)(\frac{1-\alpha_0^2}{1+f_c})$$

$$s.t. \quad \alpha_0=\frac{k_m}{2e^{(-\lambda I_0)}(\frac{1}{\lambda}+I_0-k_c)Z_0}$$

$$I_0=\frac{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2}k_c}{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2}-\exp(S\alpha_0^2)}$$

$$d_m+d_c=f_c+f_m$$

3.2.3 Monopoly Card Network

A monopoly network maximizes the card profits instead of transaction volume.

$$\underset{f_c,f_m}{Max}\ \ e^{(-\lambda I_0)}(\frac{1}{\lambda}+I_0-k_c)(\frac{1-\alpha_0^2}{1+f_c})(f_c+f_m-d_c-d_m)$$

$$s.t. \quad \alpha_0=\frac{k_m}{2e^{(-\lambda I_0)}(\frac{1}{\lambda}+I_0-k_c)Z_0}$$

$$I_0=\frac{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2}k_c}{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2}-\exp(S\alpha_0^2)}$$

3.2.4 Social Planner

The social planner maximizes the social surplus subject to the incentive constraints of merchants and consumers as well as the balanced-budget constraint:

$$\underset{f_c,f_m}{Max}\ \ e^{(-\lambda I_0)}(1-\alpha_0^2)\{(\frac{1}{\lambda}+I_0)(\frac{\tau_c+\tau_m}{1+\tau_c})-(\frac{1}{\lambda}+I_0-k_c)(\frac{d_c+d_m}{1+f_c})\}$$
$$+\frac{\alpha_0^2e^{(-\lambda I_0)}k_c}{(1+\tau_c)}(\tau_m+\tau_c)-e^{(-\lambda I_0)}k_c-(1-\alpha_0)k_m$$

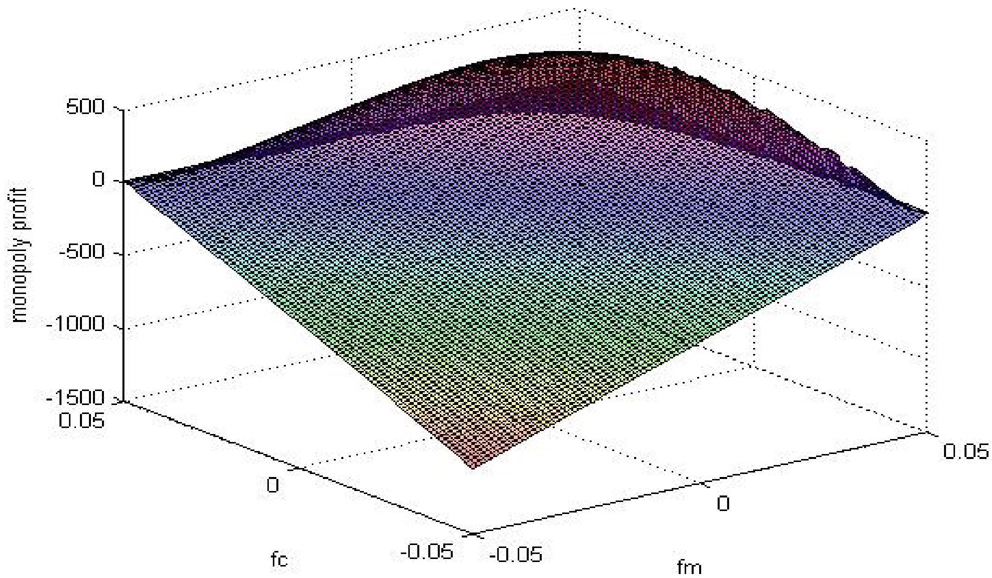

Figure 6: Card Fees and Monopoly Profits

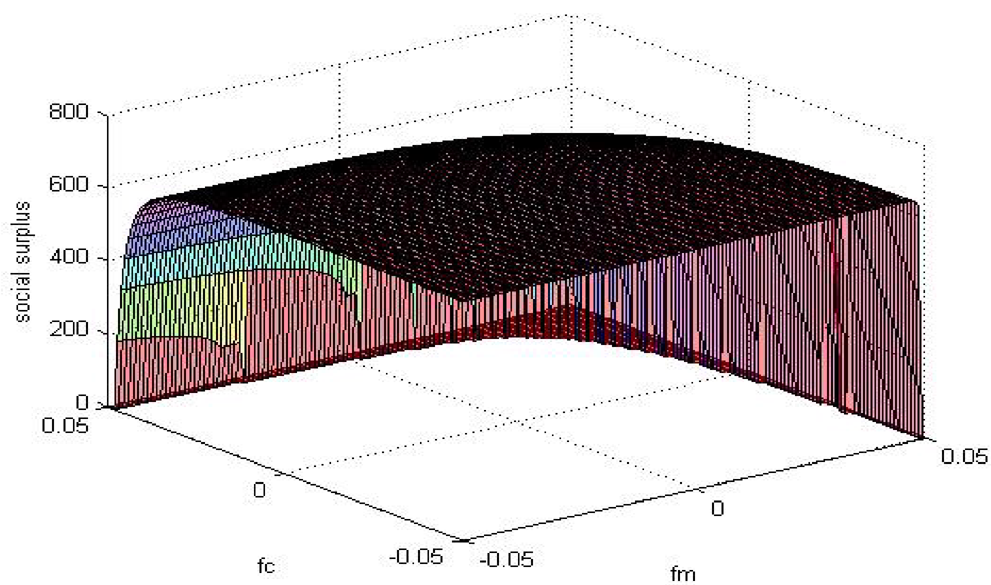

Figure 7: Card Fees and Social Surplus

$$s.t. \quad \alpha_0 = \frac{k_m}{2e^{(-\lambda I_0)}(\frac{1}{\lambda} + I_0 - k_c)Z_0}$$

$$I_0 = \frac{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2} k_c}{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2} - \exp(S\alpha_0^2)}$$

$$f_m + f_c \geq d_c + d_m$$

3.3 Findings

As shown, a competitive card network, a monopoly card network, and the Ramsey planner each solves a different problem. The resulting market outcomes and competitive dynamics are well illustrated with our simulations (see attached figures in the Appendix).

Our major findings are:

- At a given total cost $d_m + d_c$, the monopoly network chooses the highest price level, $(f_m + f_c)$, and, correspondingly, the lowest card adoption and usage; the competitive network chooses the highest interchange fees (the greatest difference between merchant and consumer prices), resulting in the highest card adoption and usage. The Ramsey planner chooses fees that are lower than the monopoly fees, and imply lower interchange fees than the competitive network, and which result in less adoption and card use than the competitive network. Because the competitive network does not take into account the adoption costs of merchants and consumers, it results in excessive adoption of cards from a social point of view.

- As the total cost $d_m + d_c$ declines, all three market structures, competitive, monopoly and the Ramsey planner, choose decreasing fees $f_m + f_c$ and generate more card adoption and transactions. However, the monopoly network has the smallest fee reduction.

- The cost allocation tends to be different under different market structures. As shown in simulation 1, with a symmetric adoption cost k_m and k_c, the monopoly charges the most similar fees to both merchants and consumers, the competitive network charges the most divergent fees to merchants than consumers, while the Ramsey planner charges fees than are intermediate, in their degree of divergence, relative to the other network structures.

  This is a key result and worth considering further. Consider the move from a competitive card industry structure to a monopoly one for a given level of total costs. We observe that the monopoly tends to raise prices more on consumers. Why is this? The intuition for this result can be understood by considering the first-order effects from the card-provider's optimization conditions. Consider, in the move from a competitive card-industry to a monopoly card-industry with a marginal increase in merchant fees and no change in consumer fees. The loss in revenue from transactions will consist of all the income from cards in the marginal store (that declines card acceptance after the price increase). Alternatively, consider a marginal increase in consumer fees and no change in merchant fees. The loss in revenue will consist of the loss from the marginal (low-income) consumer's transactions in all stores. Under the assumption of a more highly skewed distribution of income than firm size, the former loss is greater for the monopolist than the latter, so the monopolist tends to raise consumer fees relative to the competitive firm. For the move from a private organization of industry to the Ramsey planner, the planner, in addition to valuing the card transaction, values the benefits from displaced high-cost cash transactions (the alternative payment device) and, at the same time, wishes to minimize adoption costs. It tends to more heavily weight a decrease in merchant fees more than a decrease in consumer fees because by lowering the merchant's fees, which leads to the adoption of the device by additional merchants, the social planner can displace the existing cash transactions in those additional merchants by the amount of the existing users of the device (without increasing any consumer's adoption costs). This is a bigger effect, given the distribution of income and firm sizes, than would be a relative decrease in consumer fees.

- As consumer incomes rise relative to card service costs (e.g., total costs of card provision may decrease with technological progress or consumer incomes may increase with economic growth), consumer fees tend to be increased relative to merchant fees. This can be understood as attempting to achieve a higher penetration rate over time, which given our income and firm-size distributions, lead to greater use of cards than would be to lower merchant fees. With an exponential distribution of income and a uniform distribution of firm size, there is a greater mass of potential transactions, as costs fall or incomes rise, in pushing consumer, rather than merchant adoption. This result is robust under all three different market structures.

- When we examine distributions of income and firm size that are equally skewed (and both are exponential distributions), we find that card adoption and transaction volumes fall. Consider the Ramsey planner: with an equally skewed distribution of income and firm size, the planner can achieve more card transactions by setting prices to induce the larger merchants and the higher income consumers to adopt cards, while sparing the smaller merchants and lower income consumers from incurring the adoption costs of cards (relative to the case in which firm size is less highly skewed). In both the competitive and monopoly networks we find that the divergence between consumer and merchant fees increases as the size distribution of the merchant becomes more skewed and matches that of the income distribution.

- The fee allocation is influenced by the adoption cost k_m and k_c so that the party having a higher card adoption cost tends to bear a lower card service fee (as shown in simulations 2 and 3). This result is robust under all three different market structures.

- The simulations show that using identical distributions for income and firm size do not alter the results drastically. Both distributions were assumed to be exponential distributions, which are quite skewed. The general result as we change the firm size distribution from a uniform to an exponential distribution is that merchants pay higher prices (and consumers lower prices) in both the competitive and monopoly cases, resulting in fewer (but larger) firms adopting cards. Consumer adoption and card transaction volume are little changed, however. An implication is that in markets that have more highly skewed distributions of merchants, we can expect higher interchange fees. These results point out that there are fundamental asymmetries between the merchants and consumers in our model that drive, to a considerable extent, the equilibrium pricing patterns. These asymmetries are built into our economic model in that the consumer does not have a direct utility for card use, while cards are a direct input for the merchant; in other words, the asymmetries are present because of the economic roles played by consumer and merchant.

3.3.1 Applications: Interchange Fee Puzzles

Our model provides a general framework to study the pricing, adoption and usage of payment devices. When applied to the payment card, it sheds lights on several puzzles surrounding the payment card interchange fees.

First, for some payment card systems (e.g., debit cards in the U.S.), why did the interchange fees flow from consumers to merchants in the early years only to have the direction reversed more recently? More generally, why have interchange fees increased in recent years?

As total costs of card provision is decreased, as would occur with technological progress, our model suggests that fees would decrease relatively for consumers. As we explained earlier, this can be understood as attempting to achieve a higher penetration rate over time, which given our income and firm-size distributions, lead to greater use of cards than would be to lower merchant fees. With the more-skewed income distribution, there is a greater mass of potential transactions, as costs fall, in pushing consumer, rather than merchant adoption. Furthermore, our theory suggests that consumers might have to pay interchange fees in early years: early in the evolution of debit cards, there was a higher adoption cost k_m for merchants relative to consumers as merchants had to install new card terminals, while consumers were endowed with debit cards through their banks' delivery to them of ATM cards (which then could function as debit cards). Consequently, our model would suggest that consumers had to bear a larger share of the card service costs. Later on, as the merchants' adoption cost k_m declined (as general purpose credit and debit card terminals became available, and terminals fell in cost) more card cost was shifted to the merchants and the interchange fees could reverse its direction.

Second, why haven't the interchange fee paid by merchants fallen rapidly with the technology progress in the U.S. as well as worldwide?

Our theory suggests it might be explained by several factors: (1) the skewed distribution of income leads to relative price declines for consumers over time as costs of provision fall; (2) an increase in monopoly power as the card industry matures may slow down the reduction of card service fees; (3) increasing skewness in the distribution of merchant sizes may cause merchants to bear more card service costs relative to consumers. We believe that all factors matter given the observations that the concentration of payment card networks has been increasing, and at the same time, the card networks rely more and more on the consumer rebates to boost the card usage.

Another contribution of our theory to empirical issues is related to the relatively low interchange fees charged to grocery stores and gas stations in the U.S. relative to department stores. We would suggest that grocery stores and gas stations suffer higher adoption costs of accepting electronic payment cards relative to department stores, for example, as grocery stores and gas stations must install terminals in many more locations per dollar of sales when compared with department stores. As those particular merchants' adoption costs are higher, the consumer bears a higher share of the payment card costs in those venues than in the department stores.

4 Final Remarks

We have provided an alternative microstructure of two-sided markets to study the pricing, adoption and use of payment devices, in which we emphasize the roles that consumers' income distribution and merchants' size heterogeneity play in adopting new payment devices. Unlike many existing studies, we assume a competitive economy where both merchants and consumers behave nonstrategically. The richness of the strategic approach is sacrificed in favor of a focus on the interplay between individual firm and consumer decision-making and aggregate industry characteristics. As a result, our model provides a convenient framework to study evolution of the payment industry both in the short run (network "chicken-egg" dynamics) and long-run (dynamics due to cost and income changes), and offers some further insights into the related competition policy issues.

Our analyses show that consumer income, adoption cost, and market structure each play important roles in determining the pricing and usage of payment devices. In particular, when applied to the payment card industry, our findings suggest that both the increasing concentration of payment card networks, decreases in the costs of adopting payment cards by consumers, and increasing skewness in the firm size distribution may help explain the puzzles surrounding interchange fees. Furthermore, our model focuses particular attention on adoption costs in explaining the dynamics of the direction of interchange fees in the U.S. debit card industry, and in the array of interchange fees chosen by card networks in different industries.

We suggest several extensions of the model. First, we may introduce sunk costs of adopting the payment devices for merchants and consumers to further address the dynamics of adoption and pricing. Second, we may model explicitly the competition among payment networks.

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Appendix

Proof of Equation L2:

Recall Equation 10:

$$I_0 = \frac{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2} k_c}{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2} - \exp(2\int_{\alpha_0}^{\frac{Z_0}{Z_1}\alpha_0} \alpha \ln(\frac{(1-\tau_m)\alpha}{(1-f_m)\alpha-(1+f_c)\alpha_0 Z_0}) d\alpha)}$$

Note that

$$\begin{aligned} & \int \alpha \ln(\frac{(1-\tau_m)\alpha}{(1-f_m)\alpha - (1+f_c)\alpha_0 Z_0}) d\alpha \\ = & \frac{\alpha^2}{2}(\ln(1-\tau_m)) - \frac{1}{2}\alpha^2 \ln((1-f_m) - \frac{\alpha_0 Z_0(1+f_c)}{\alpha}) \\ & + \frac{\alpha_0 Z_0(1+f_c)}{2(1-f_m)}\alpha + \frac{\alpha_0^2 Z_0^2(1+f_c)^2}{2(1-f_m)^2}\ln(\alpha - \frac{\alpha_0 Z_0(1+f_c)}{(1-f_m)}) \end{aligned}$$

Therefore, we derive

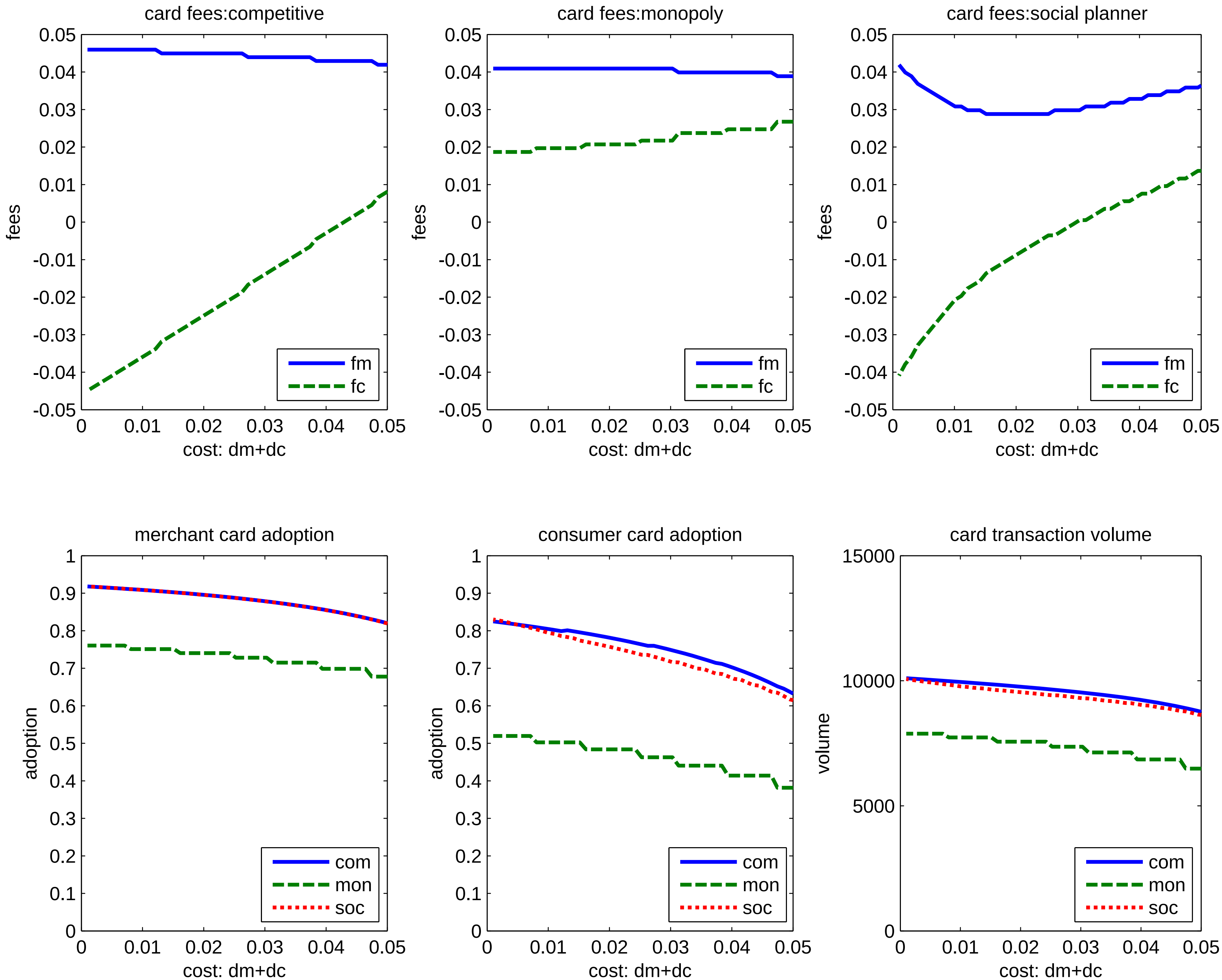
$$\begin{aligned} & \int_{\alpha_0}^{\frac{Z_0}{Z_1}\alpha_0} \alpha \ln(\frac{(1-\tau_m)\alpha}{(1-f_m)\alpha - (1+f_c)\alpha_0 Z_0}) d\alpha \\ = & \frac{\alpha_0^2 Z_0^2(1+f_c)}{2(1-f_m)Z_1} + \frac{\alpha_0^2 Z_0^2(1+f_c)^2}{2(1-f_m)^2}\ln((\frac{Z_0}{Z_1}\alpha_0) - \frac{\alpha_0 Z_0(1+f_c)}{(1-f_m)}) \\ & + \frac{1}{2}\alpha_0^2 \ln\frac{(1+f_c)}{1+\tau_c} - \frac{\alpha_0^2 Z_0(1+f_c)}{2(1-f_m)} - \frac{\alpha_0^2 Z_0^2(1+f_c)^2}{2(1-f_m)^2}\ln(\alpha_0 - \frac{\alpha_0 Z_0(1+f_c)}{(1-f_m)}) \\ = & \frac{1}{2}\alpha_0^2 \ln\frac{1+f_c}{1+\tau_c} + \frac{\alpha_0^2 Z_0(1+f_c)}{2(1-f_m)}(\frac{Z_0}{Z_1} - 1) + \frac{\alpha_0^2 Z_0^2(1+f_c)^2}{2(1-f_m)^2}\{\ln(\frac{(\frac{Z_0}{Z_1}) - \frac{Z_0(1+f_c)}{(1-f_m)}}{1 - \frac{Z_0(1+f_c)}{(1-f_m)}})\} \end{aligned}$$

Equation 10 can then be rewritten into

$$I_0 = \frac{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2} k_c}{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2} - \exp(S\alpha_0^2)} \tag{L2}$$

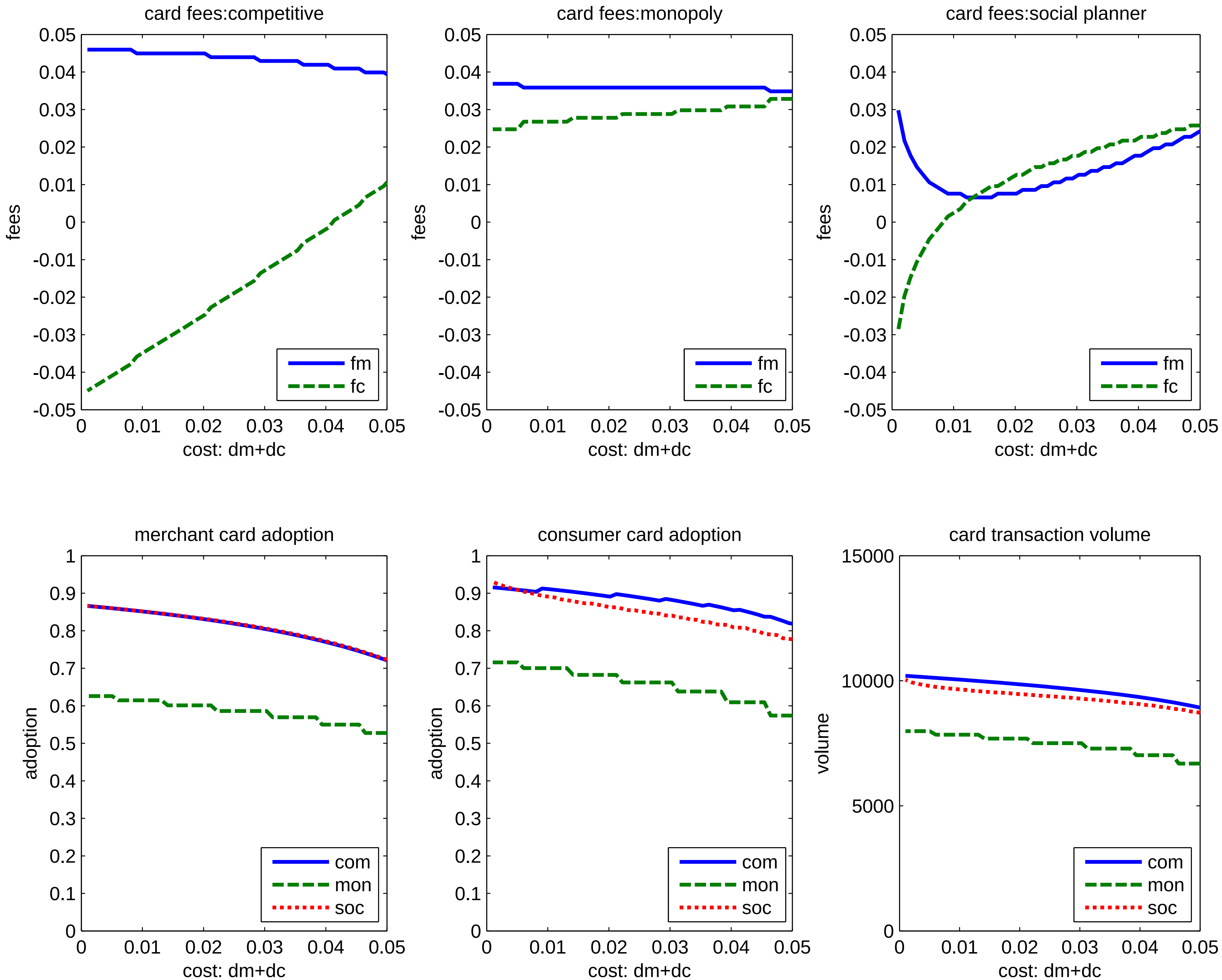
with $S = \ln\frac{1+f_c}{1+\tau_c} + \frac{Z_0(1+f_c)}{(1-f_m)}(\frac{Z_0}{Z_1} - 1) + \frac{Z_0^2(1+f_c)^2}{(1-f_m)^2}\ln(\frac{\frac{Z_0}{Z_1} - \frac{Z_0(1+f_c)}{(1-f_m)}}{1-\frac{Z_0(1+f_c)}{(1-f_m)}})$.

Simulation: Uniformly Distributed Merchant Size and Exponentially Distributed Consumer Income



Lamda=0.0001, Km=150, Kc=150, Tm=0.05, Tc=0.05

Simulation: Uniformly Distributed Merchant Size and Exponentially Distributed Consumer Income



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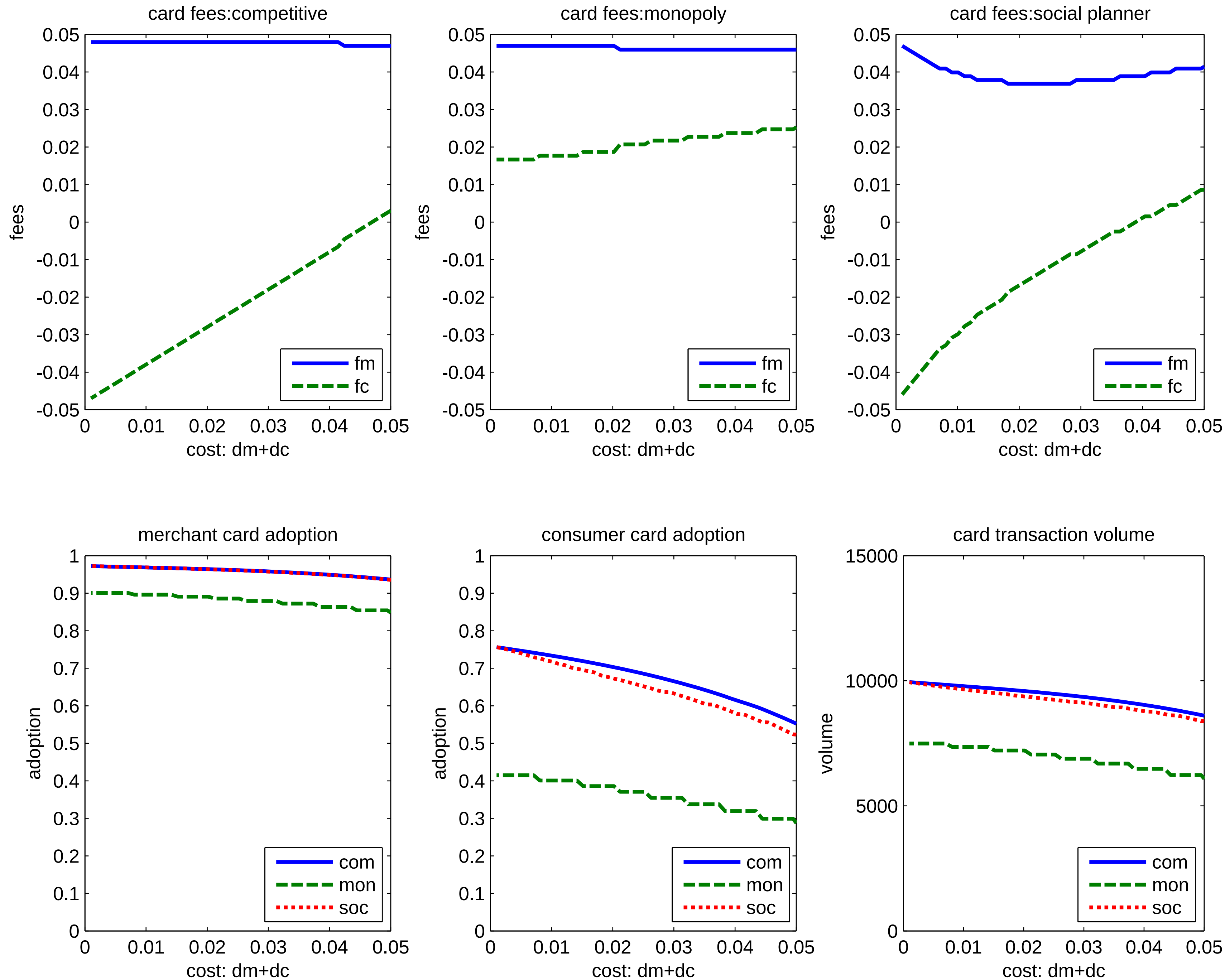
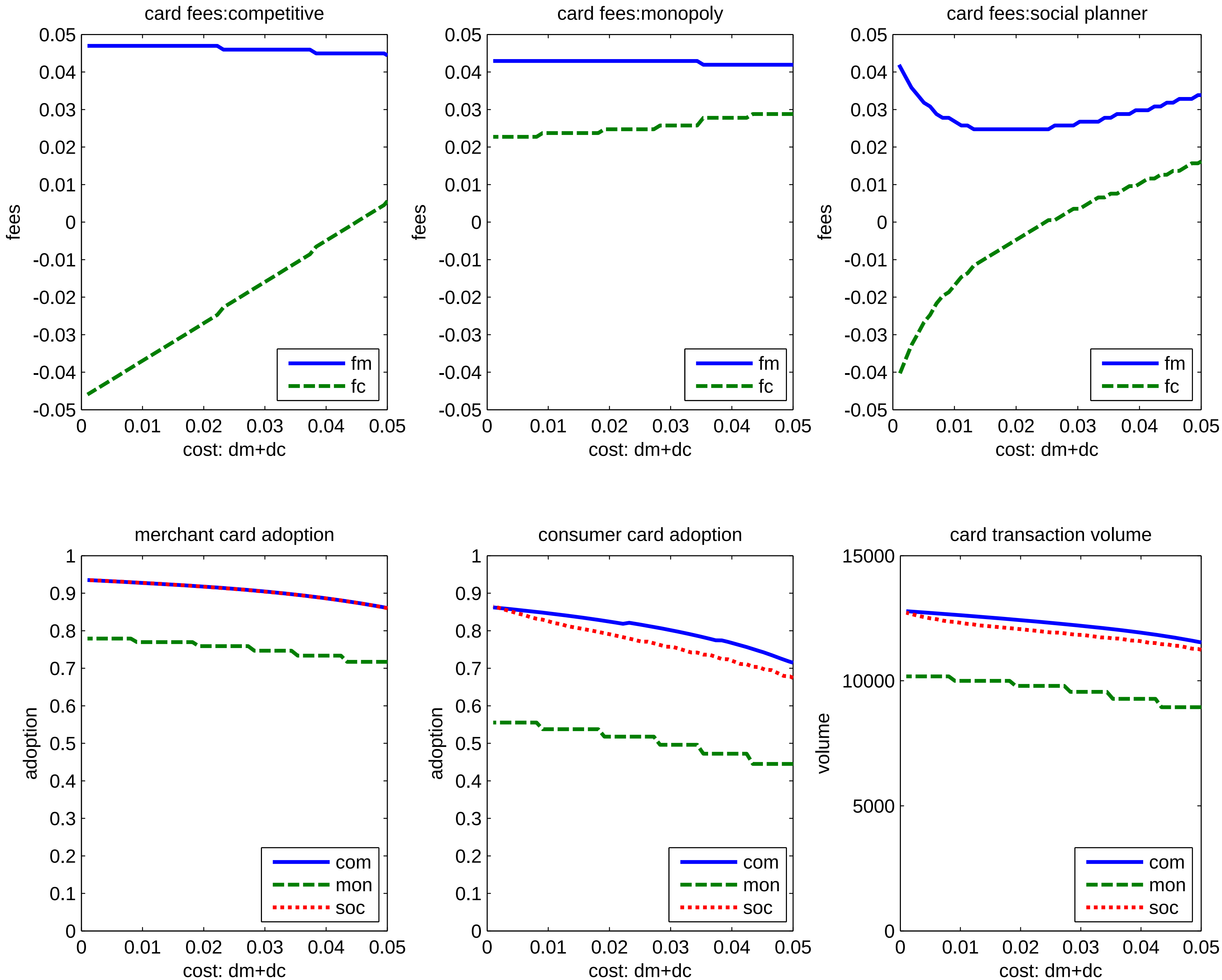
Simulation: Uniformly Distributed Merchant Size and Exponentially Distributed Consumer Income



Lamda=0.0001, Km=50, Kc=250, Tm=0.05, Tc=0.05

Simulation: Uniformly Distributed Merchant Size and Exponentially Distributed Consumer Income



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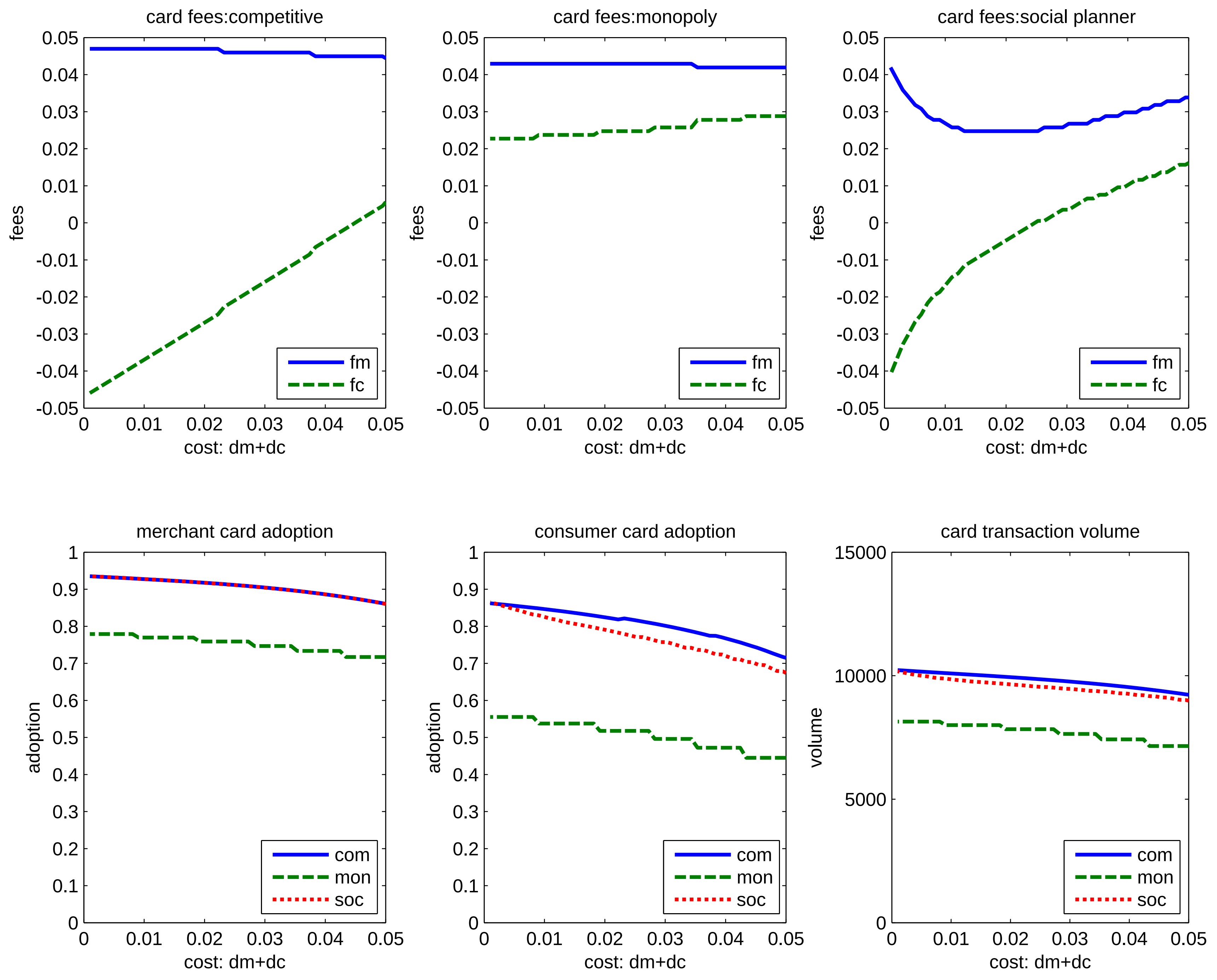
Simulation: Uniformly Distributed Merchant Size and Exponentially Distributed Consumer Income



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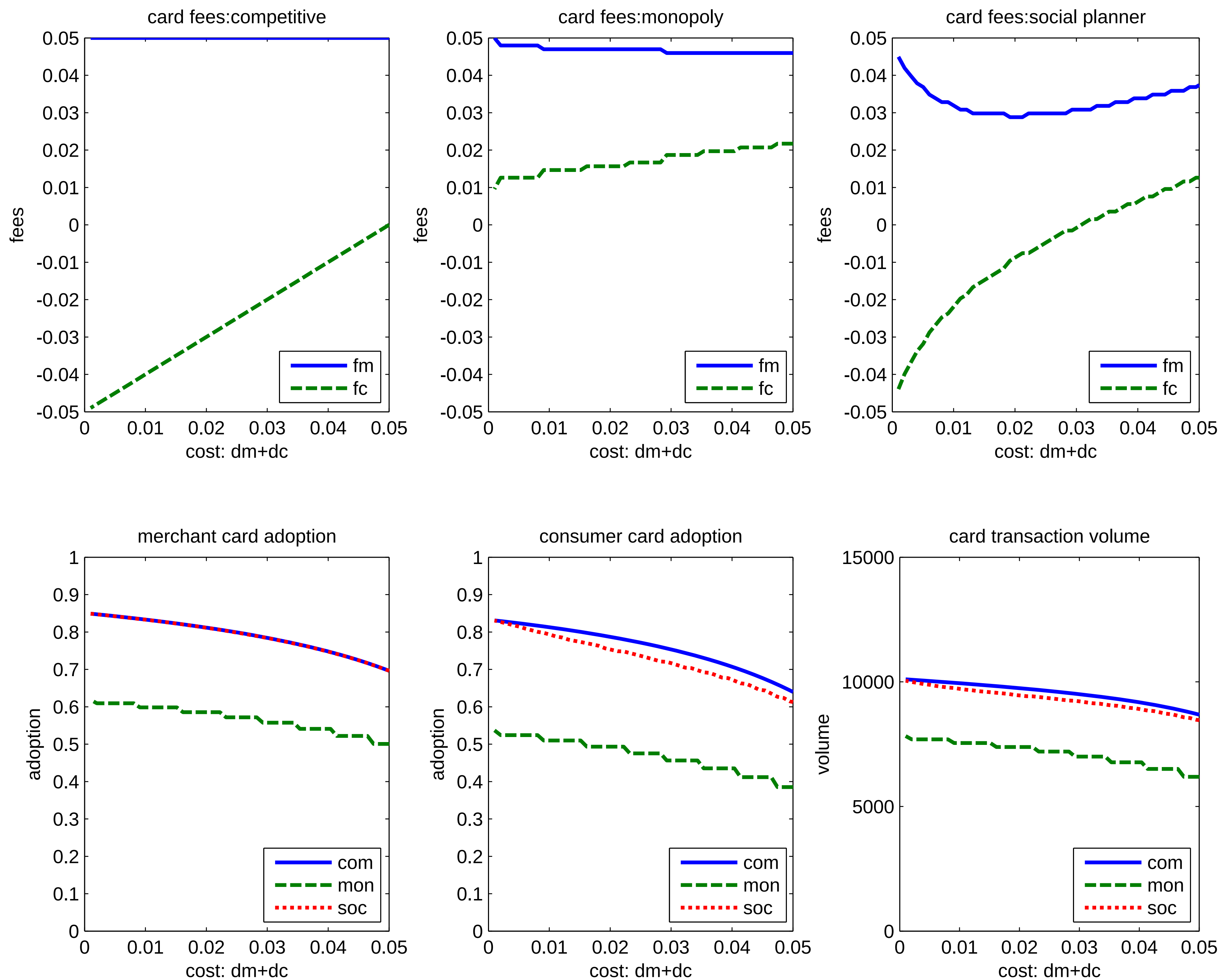
Simulation: Exponentially Distributed Merchant Size and Exponentially Distributed Consumer Income



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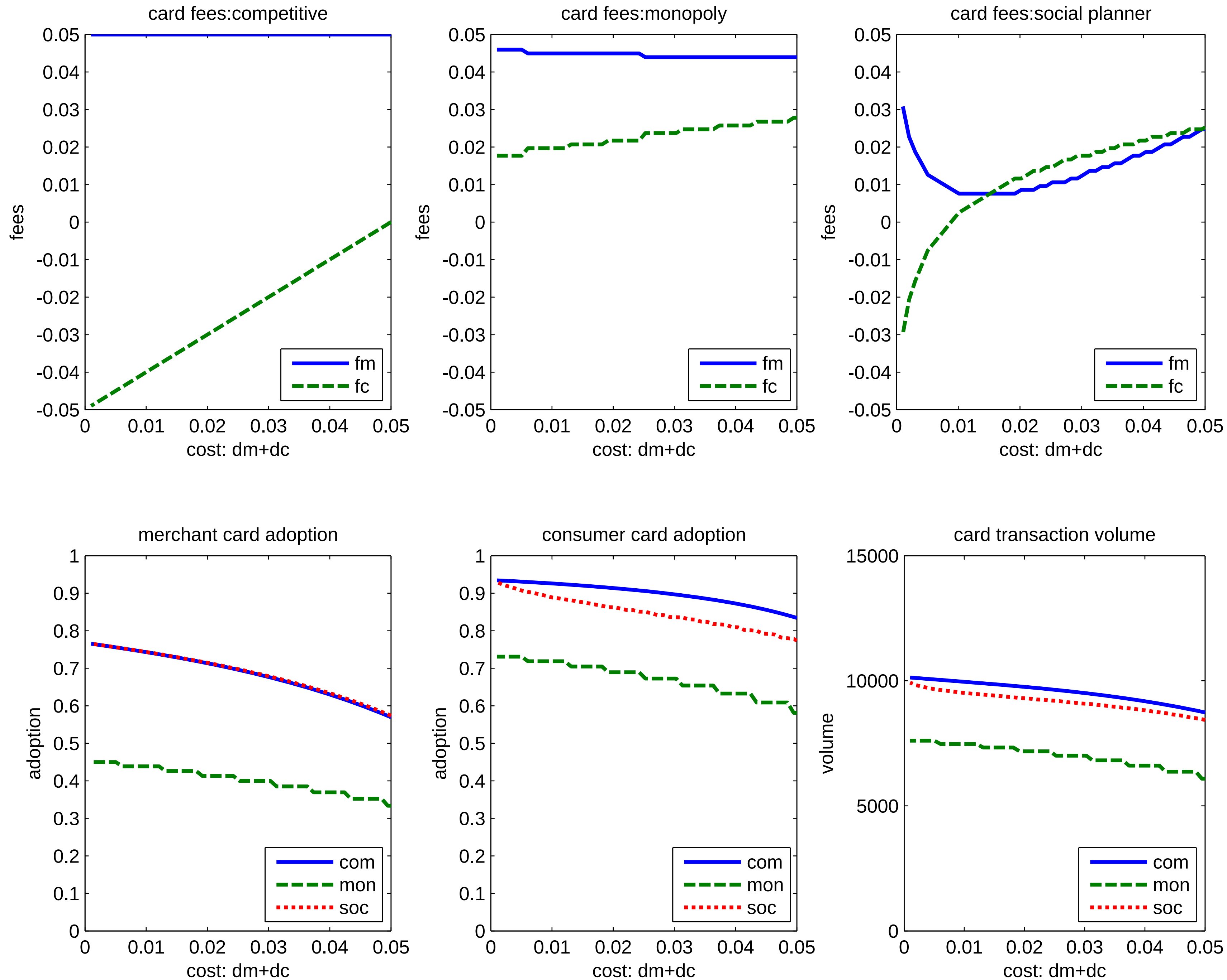
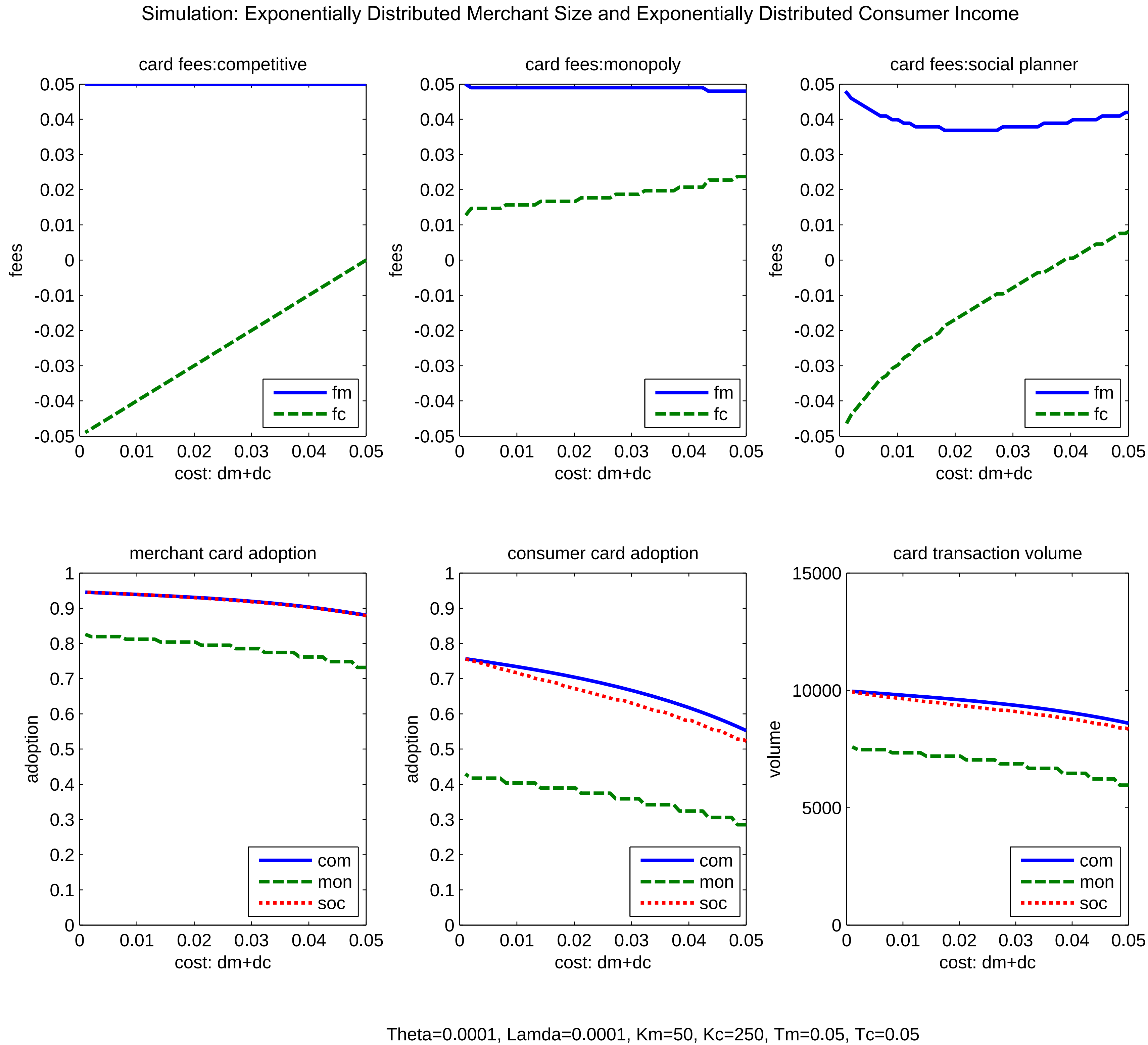
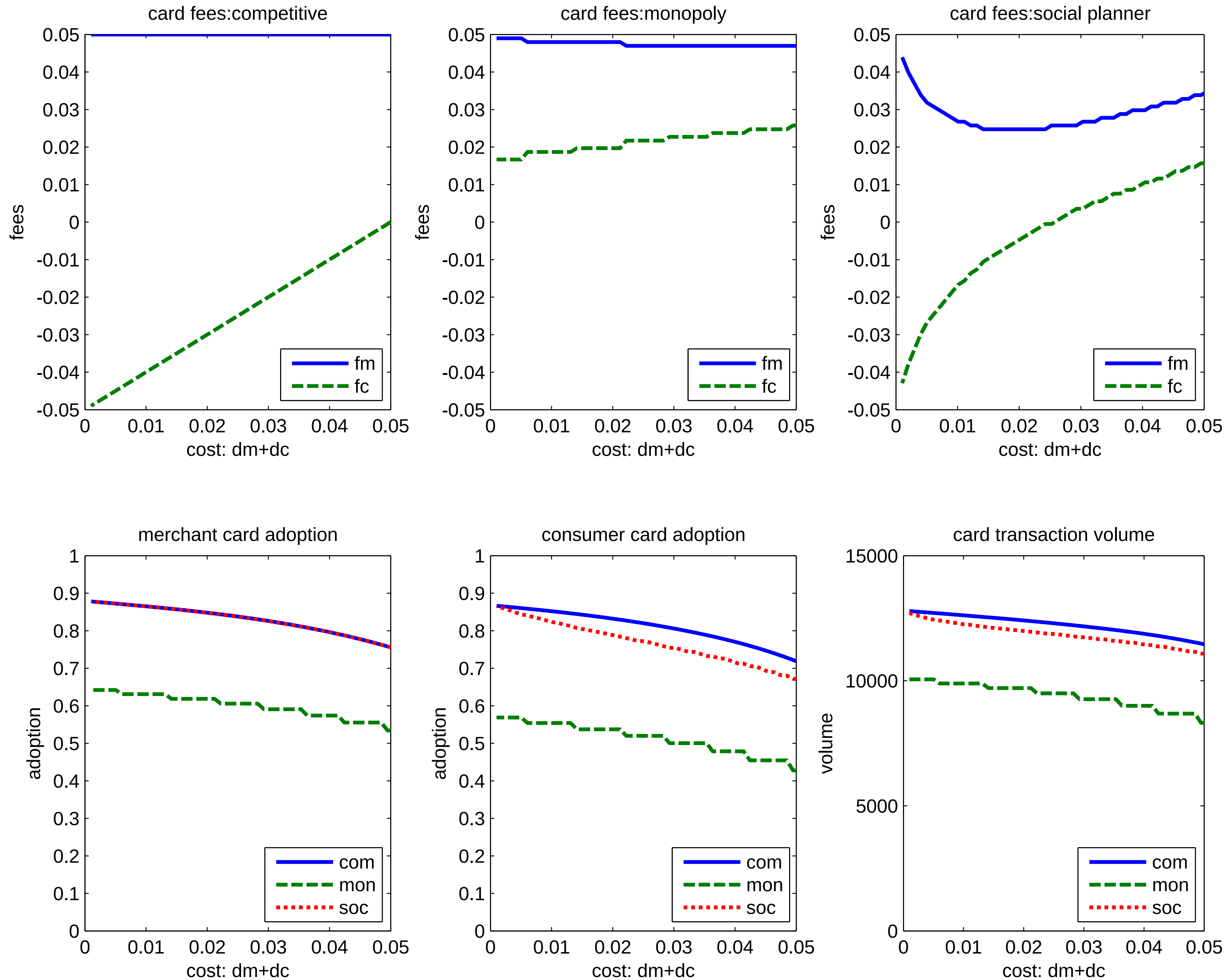
Simulation: Exponentially Distributed Merchant Size and Exponentially Distributed Consumer Income



Theta=0.0001, Lamda=0.0001, Km=250, Kc=50, Tm=0.05, Tc=0.05

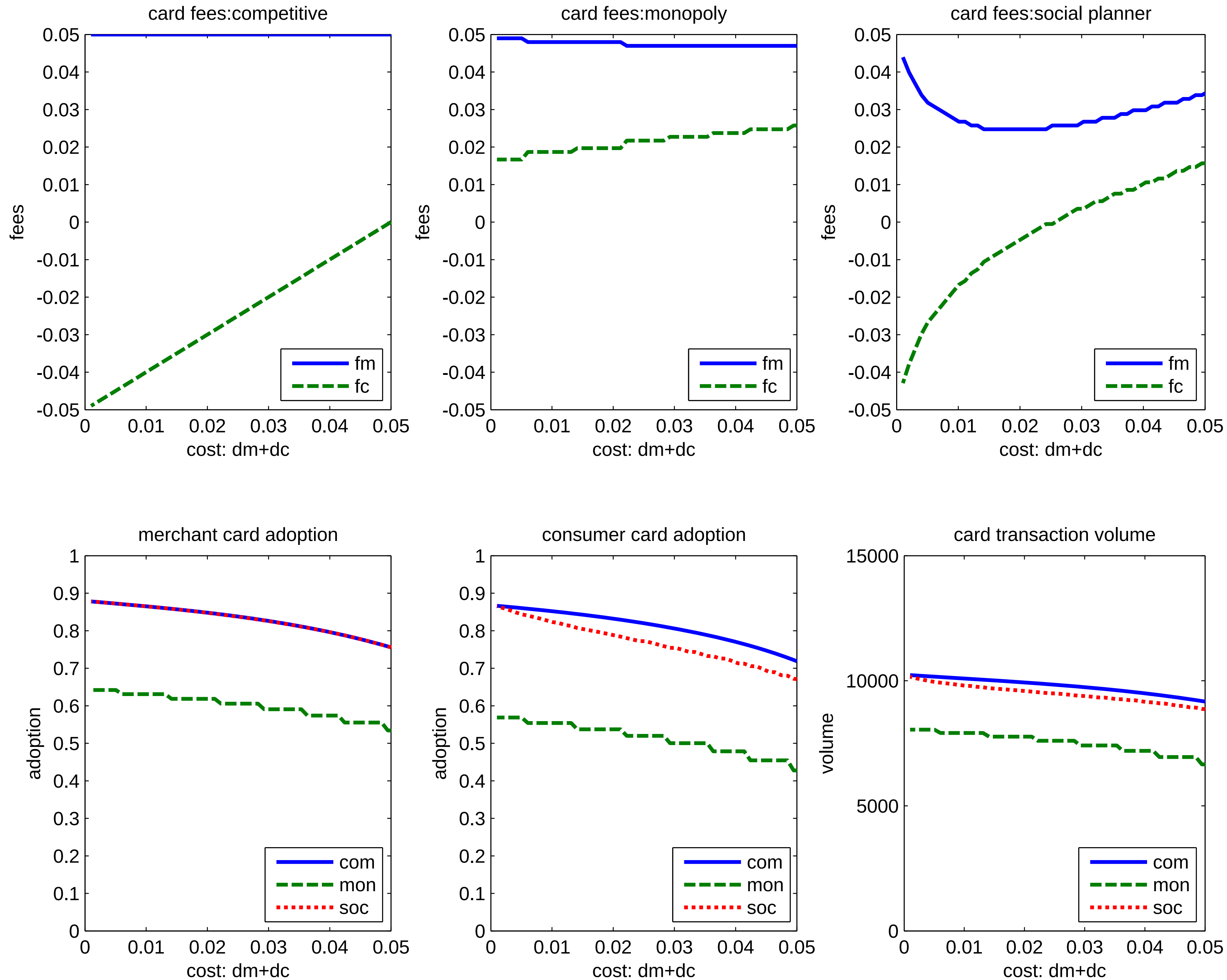

Simulation: Exponentially Distributed Merchant Size and Exponentially Distributed Consumer Income
card fees:competitive
card fees:monopoly
card fees:social planner
fees
cost: dm+dc
fm
fc
merchant card adoption
consumer card adoption
card transaction volume
adoption
volume
com
mon
soc
Theta=0.0001, Lamda=0.0001, Km=50, Kc=250, Tm=0.05, Tc=0.05

Simulation: Exponentially Distributed Merchant Size and Exponentially Distributed Consumer Income



Theta=0.0001, Lamda=0.00008, Km=150, Kc=150, Tm=0.05, Tc=0.05

Simulation: Exponentially Distributed Merchant Size and Exponentially Distributed Consumer Income



Theta=0.0001, Lamda=0.0001, Km=120, Kc=120, Tm=0.05, Tc=0.05